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What Professional Knowledge and Experiences Should the Teacher Training Program Provide?

By C. N. Mills
Illinois State Normal University, Normal, Illinois

The general theme of the various sections of the Council centers around the student who is not going to college after high school graduation. This idea that many students will not go to college might apply to certain highly industrialized communities or to students who are classified as the below average group. I am of the opinion that the non-college student will soon be a condition of the past. A few years ago about 1 out of 20 went to college; latest reports say that the ratio is about 1 to 9; however today it might be around 1 to 5. What is making for all of this change? Many factors have their contributing influences. Today we hear about the junior college, or closely related post-high school plan; the Night School; Correspondence Study; the NYA Movement; and the CCC camp development. Colored by all of these movements we have the students facing the problem of unemployment. Hence there seems to be only one general course for him to continue, and that is he must take up some phase of advanced training.

For students who are classified as average or superior the continuance of advanced training means that certain basic areas of learning must be met and mastered. We learn about students, superior students too, who are returning from higher institutions of learning to the local high schools to take over courses in science and mathematics so as to gain a working knowledge of the various subjects. This, from one point of view is a wholesome situation, as the report soon gets to other students about this humiliating condition, and the school officials are then willing to offer the usual courses so necessary in the field of algebra and geometry.

Mathematics in the various compartments as arithmetic, algebra, and geometry, has been receiving a great amount of criticism during the past few years. As a result we find about thirty-five of the States removing the algebra and geometry from the required subjects for graduation. I can see this movement as a beneficial setup, for under such conditions a real challenge is placed at the door of the teacher. To the real teacher of mathematics it means an improvement in the teaching in order that students will receive a greater appreciation and understanding of the great values of mathematics. This point of view brings me directly to the thesis that I wish to present.

We can talk, discuss, recommend, make reports, attend professional meetings, and argue until we are black and blue in the face, yet all of these will not cause the at-

titude toward algebra and geometry to change. The change will take place only through the action and demands of the high school pupils themselves. Once the pupils are inspired, the influence with parents, school boards, and administrators, will crystallize in the offering of more mathematics. The direct demands are always more effective than superficial theories. We have learned about such changes taking place in many communities. A new renaissance is taking place relative to mathematics as one of the important factors in the continuance of civilization. We must do our part to stem the tide which would remove the basic training of our youth, or the day will come when communism will rule supreme.

Naturally, you will ask "How can we stem the tide?" I have just one answer to offer at this time. Give me a real teacher of mathematics, not a person who is ruled by the textbook, and the youth will make pathways to his door. A teacher of this type must have a broad training, good understanding of human behavior, and plenty of common sense, if he is to engender the interest and appreciation of the basic values of mathematics. Yes, curiosity has its worth if it will lead the pupil on to seek more truth.

Since our discussion is concerned with the field of geometry, I wish to point out that a teacher of high school mathematics should have some knowledge of the college geometry and at least an introduction to projective geometry. Applying some of the great fundamental principles of these two fields of geometry by means of model constructions, drawings, and paper folding, offers a rich mine of inspiration when the proper guidance is set up. When the pupil sees with his own eyes the beauty of geometrical continuity, symmetry, and duality, he is at once captivated by their mysteries. He then is won into the fold of learning when he says "Behold! Why?" Such interest and desire for more light can be guided only by a real teacher of mathematics.

I would not neglect the algebraic analysis in the courses in advanced geometry. It gives the pupil much satisfaction and assurance when he can see his conclusions justified by the means of symbolism. Also by the use of algebraic analysis he is led into the theory of possible and impossible constructions. Take for instance the three great problems of antiquity. Think of the research that has been stimulated by those problems. Furthermore, the pupil will soon learn that algebra and geometry are only compartments in the broad fields of learning. Such a relationship will naturally lead us into the general topic of number systems, but I shall not delve into this topic now.

Thus far the discussion has emphasized the cultural values of mathematics as a means to cause the pupil to do reflective thinking. When this goal is attained, the utilitarian value can be stressed and thus the pupil will realize that his courses in mathematics present a challenge for best efforts to be put forth.

Field trips to the many types of industrial establishments will bring to the students a greater appreciation of the uses of mathematics. One of the most interesting applications comes in the sheet metal layout industry. Even here the interesting properties of descriptive geometry are needed. In the field of design as used in architecture and art we find plenty of material that spurs on the pupil's interest. And above all, what about the geometry of nature? I think we can truthfully say as Plato did centuries ago "God Geometrizes." An interesting experiment is to construct the Caustic curves of reflection and refraction. A speaking gallery offers most interesting queries. The construction phases of Involutes and Evolutes, Locus of Centers of Curvature, Envelopes of variable lines, Harmonic Ranges and Pencils, Inverse Figures, Linkages, Pole and Polar, Involutions, Pascal and Brianchon Theorems, Desargues Theorem, all of these have their place in the guidance of the high school pupil into the realm of mathemat-

ics. Once the pupil has been inspired and curiosity aroused he will have the real spirit to "carry on." There is plenty of fun ahead for the individual who likes to solve and create new problems.

The fields of college geometry and projective geometry possess a mine of inspirational materials which when properly used makes our teaching a pleasurable task. Think of the vast storehouse of knowledge between the lines of the textbook, and this guidance into the unseen is the task of the teacher. Start the pupil on his way rejoicing, and mathematics will be recognized in the proper light.

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Training Teachers of General Mathematics

By C. H. Butler
State Teachers College, Kalamazoo, Michigan

Any discussion of the training of teachers of general mathematics must be approached, I think, through a consideration of what the job implies. This, in turn, involves an examination of the nature and status of the subject and of the reasons for teaching it. It seems appropriate, therefore, at the outset to try to arrive at some acceptable understanding of the meaning of general mathematics and of its place in the educational program.

General mathematics is a well-known but not a very well defined term. There have been associated with it many shades of meaning and interpretation. Within its broad category courses of study and textbooks have been variously designated by such titles as "correlated mathematics," "fused mathematics," "integrated mathematics," "practical mathematics," "social mathematics" and so on, and each of these titles carries its own special connotations. The one thing which they have in common is that they all represent departures from the so-called traditional courses in which arithmetic, algebra, and geometry were highly segregated. That is to say, they all represent efforts to bring out values held to be more or less submerged in the traditional courses but believed to inhere in certain reorganizations of subject matter and reallocations of emphasis.

There is no complete agreement as to the detailed content and organization of these courses. Authors of textbooks certainly hold different views on this point, and neither the National Committee in 1923 nor the Joint Commission in 1939 ventured to make any except broad genereral recommendations. At the same time there may be identified two rather distinct tendencies in arrangement which should be of interest to us. In some textbooks and courses of study the emphasis has been mainly upon the internal relationships of the various branches, while in others this aspect has been relatively less prominent than the practical and social bearing of the material. Now the importance of both of these aspects would be readily admitted by everybody. The differences which I have mentioned are in the *relative* importance which different authors and committees have seemed to attach to one or the other. In some cases the emphases have been quite pronounced, while between the extremes are to be found various degrees of compromise. It would seem that the most justifiable and defensible interpretation of general mathematics is one which will recognize the desirability of emphasis *both* upon the extension of theory through the natural or organic relationships of the various branches and upon the application of the subject to practical life situations. Such an interpretation is in harmony with the principles of general education, of sound mathematics and of sound learning. Therefore in the present discussion the term will be understood to imply a fair balance of emphasis in these two directions.

The movement for general mathematics is rooted in the same general causes which have operated to bring about the whole reorganization of secondary education. General mathematics has come into being as a response to feeling of need, and as an attempt to make adjustment to an educational situation differing fundamentally from that of thirty or forty years ago. The great expansion of the secondary school, the evolution of the junior high school, and certain other external conditions have been favorable to its development. At the same time there have been some factors, both external and internal, which have operated to impede its progress.

The success which has attended the general mathematics movement has not been uniform. It has been singularly note-

worthy in the junior high school, especially in the seventh and eighth grades. Ample evidence of the propriety and adaptability of general mathematics to this level of the secondary school is found in the extent to which it has established itself in these two grades. In the ninth grade it has not gained the same foothold that it has in the seventh and eighth grades. There is no doubt that college entrance requirements are partly responsible for this but not entirely. For various reasons first year algebra has not seemed to lend itself so fully either to correlation or to immediate practical application as have arithmetic and informal geometry.

A considerable number of schools in the past few years have offered a species of general mathematics in what are sometimes called "terminal courses" in the ninth grade. These courses purport to consist mainly of useful applications and extensions of the work of earlier grades rather than of any systematic development of new mathematical concepts and theory. The immediate appeal of these courses and their propriety for pupils of low ability are offset to a degree by their evident limitations with reference to subsequent work. I have no data to show just how prevalent these courses are, but it is certain that algebra still remains the typical offering in the ninth grade, even in many courses which are *called* "general mathematics."

Efforts to organize the mathematics of the senior high school into general courses have been, on the whole, unsuccessful up to the present time. This is perhaps to be expected because all fields tend to become progressively specialized at the upper levels. Demonstrative geometry and intermediate and advanced algebra continue to represent the basic offering, and there are fundamental differences between algebra and demonstrative geometry which impede efforts to effect a genuine and thorough-going fusion of the two. At the same time neither lends itself so fully to IMMEDIATE practical application as does the arithmetic and informal geometry of the junior high school. Thus in the senior high school work there are substantial blocks in both of the directions which generalized courses have tended to take. It may be noted, however, that there is an emerging movement toward general mathematics in the first year of the junior college.

I have set forth these considerations as a background against which to project a discussion of the *training of teachers* of general mathematics teachers. Certainly I cannot hope to bring you any authoritative solution of the problem, but there are certain proposals which I feel have a measure of validity and usefulness. I should like to preface these, however, by raising a question.

I think we should first ask whether the PREPARATION of the teacher of general mathematics should differ from the preparation of teachers of non-general courses, and if so, in what respects. At first thought one would probably conclude that there *should* be significant points of difference, but I have a feeling that this position is open to question. What I mean is this: as a rule any young person who plans his education with a view to becoming a mathematics teacher does so without knowing in advance whether he will be called upon to teach general mathematics or traditional arithmetic, algebra, and geometry. This being the case, it would be unsafe and unwise for him to neglect any part of his education that might have important bearings upon his future fitness for either job. I am therefore of the opinion that the basic training of teachers of general mathematics should not be narrowly conceived as a thing apart, say, from the basic training of teachers of algebra and geometry. It is my judgment that the preparation of teachers of general mathematics should include all the elements which can properly be regarded as fundamental to the training of teachers of the specialized courses, and I am inclined to think that the converse is true as well.

What, then, shall be the elements of this program?

First, I hold it to be axiomatic that every person who assumes the responsibility of teaching mathematics, general or otherwise, in the secondary school should have a thorough, systematic, and sequential grounding in the subject matter of mathematics. This subject matter preparation should include a substantial margin beyond the material which the individual expects to be called upon to teach. As a minimum it should involve a mastery of the usual academic branches up to and including a substantial acquaintance with the calculus. This is a sort of ideal to which I know we would all subscribe, but in point of fact there are innumerable situations in which this condition does not obtain. There is a feeling in certain quarters that general mathematics implies much breadth but little substance, and that the teacher of general mathematics, being a "generalist," has no need for study of the sequential specialized branches. Such a view shows a misapprehension of the real spirit in which general mathematics is to be conceived. General courses, whether pitched to the tempo of the slow pupil or not, need the strength and balance which can be supplied only by sound, extensive, and specialized scholarship on the part of the teacher.

My second principle is that the academic preparation of the teacher of general mathematics should have breadth as well as depth. It should give him more than a casual acquaintance with the *history* of elementary mathematics, with the internal relationships and interplay of the different branches, and with the relationship and application of mathematics to other fields. To this end such cognate courses as astronomy, surveying, physics, mechanics, statistics, economics, the mathematics of investment and insurance, descriptive geometry, and technical drawing can make marked contributions. It is true, of course, that much of this material cannot be incorporated directly in the secondary school courses. In many cases, however, counterparts of the advanced courses exist at elementary levels, and the teacher should be in a position to recognize these and to capitalize them where this can be done to advantage.

Third, I believe that an acquaintance with the simpler mathematical instruments and their use is important and should find a place in the preparation of the teacher. It would seem that this is of particular importance to teachers of general mathematics. These instruments, some of which can be made by the pupils themselves, can be used to advantage even in the junior high school courses where most of the general mathematics is taught, and their use adds a great deal of interest and meaning to the work at this level. The fact that some fifty teacher training institutions now offer courses along this line indicates a growing recognition of the importance of this element in the training of mathematics teachers.

Fourth, I believe that every prospective teacher of general mathematics should have some training in both freehand and mechanical drawing. To some of you this may appear to be a trivial detail, but it is certain that most pupils are visual-minded. They grasp explanations, comparisons, and illustrations most readily when these are presented graphically or pictorially or by means of models. Now a great many situations in elementary mathematics can be so represented. Some of them must be. The teacher who is unable to represent the data and relationships of problem situations by quick, clear, and accurate diagrammatic sketches will find himself working under a heavy handicap. His explanations will lack the tangibility and concreteness which immature minds demand. Obviously people are endowed with different degrees of talent and experience in drawing. Some will need less training than others. But in my opinion every prospective teacher of general mathematics ought to assure himself of at least reasonable proficiency in rapid diagrammatic drawing and in the simple use of drawing instruments. If training is needed, that fact should be recognized and the training incorporated in the program of preparation.

Fifth, I believe that through professional courses, observation, and practice teaching a deliberate effort should be made to familiarize the prospective teacher with representative instructional *details* of general mathematics. I do not minimize the importance of dealing with broad general considerations and points of view, but it seems to me that we sometimes are too reluctant to come down to earth and deal with specific matters of detail. These form the *immediate and urgent* problems with which the beginning teacher is faced, yet in some of our professional courses we never get to them. Of course it would not be possible to study all the detailed problems associated with all the topics of elementary mathematics, but I think it would be possible to set up flexible yet characteristic *patterns* for analyzing detailed teaching problems and to give specific training along this line. It is my opinion that at least a reasonable balance should be maintained between these direct instructional problems and the more general considerations.

Sixth, I feel that prospective teachers of *general* mathematics ought to be made aware of, and familiar with, effective and appropriate means of stimulating and maintaining interest in the work. It would be hard to overemphasize the importance of this. Lack of real interest is probably responsible for a great deal of the mediocrity which marks the achievement of so many pupils, but pupil interest can often be greatly enhanced by appropriate efforts on the part of the teacher. The practical applications and social uses of mathematics appeal strongly to most pupils, and there is now available a considerable literature describing and illustrating such applications. Anecdotes and sketches drawn from the history of mathematics and science can do much, if rightly presented, to enrich the pupils' appreciation of the subject. Similarly, by showing the need for mathematics in other studies or lines of work, by bringing out the interrelations of different mathematical fields, by playing up the element of curiosity, by illustrating the meaning and nature of deductive thinking, by the use of mathematics recreations, exhibits, club programs, etc., and by other means appropriate to the ages and interests of the pupils much can be done to add zest to their work. However, unless the teacher has knowledge of these things and knows how and where to find helpful suggestions he will be handicapped in his efforts to raise and keep the level of interest to a point where it will have an appreciable effect upon the character of the work. Therefore I feel that in connection with the professional courses this matter of arousing and maintaining interest should receive considerable attention.

The six points which I have mentioned obviously do not form an exhaustive catalog, but they do represent considerations which seem to me to have significant and direct bearing upon our problems. If I were to add a seventh, it would be a plea for an adequate general education to serve as a supplement for the teacher's technical preparation and, with that, as a base for his professional training. This has already been more or less implied, but I feel that it is important enough to deserve special mention. You will say, of course, that such a background should be a requisite for all teachers, and you will be right. But that does not make it any the less a requisite for teachers of general mathematics. Indeed, the very flexibility of the subject matter in this field suggests that the specific need for a broad cultural foundation may be even more evident and more acute here than in some of the more specialized courses. It is being recognized more and more clearly that specialization which is too narrow and professional training that begins too early do not produce the most adequate teacher. I feel that if the program of training for teaching general mathematics is to yield its maximum benefit, the professional training should *presuppose* the necessary technical study, which in turn should be woven as a dominant pattern through an enriching fabric of general culture and understanding for all concerned.

Professional Preparation of Teachers of Algebra*

By A. R. Congdon
University of Nebraska, Lincoln, Nebraska

Since just two of us have been assigned to the task of discussing the training of teachers of algebra, I presume I am justified in assuming that any phase of this training which has not been covered by Professor Hassler is a legitimate topic for me to discuss. Thus, you see, I am avoiding the necessity of defining professional training.

A few years ago I had occasion to examine the minutes of the North Central Association beginning about 1900. Even at that time they were insisting upon reforms in mathematics. Then, as now, the content was being severely criticized. Not until a much later date did anyone have the courage to suggest that the manner of presentation might be improved.

We, who began teaching 35 or 40 years ago, were prepared for that work by taking all of the college courses in mathematics that were offered, both under-graduate and graduate, with no courses whatever of the type now known as educational courses. The various associations of mathematics teachers in the United States and England now favor some type of professional training, but are not agreed upon the amount or the type of training. A quotation in the Fourteenth Year Book from the *Mathematical Gazette* (December 1919), seems to be somewhat in advance of our thinking at that time concerning the training other than in mathematics for the teachers of mathematics: The aim of professional training is "not merely to turn out a competent craftsman, but to form a young man or a young woman into an enlightened member of a body that has an enormous responsibility for the well being of a nation. While he (the young graduate) is still warm with the generous and universal spirit that university life should have awakened in him, he should be led to inquire into the meaning of education and to understand something of the significance in relation to the many-sided business of life. He should learn how much wider education is than mere teaching, and should gain inspiration and right direction from those who have reflected most deeply upon it. . . . And he should have his bias as a specialist corrected by observing how all the major subjects of the curriculum answer to deep seated needs of the human spirit and represent essential currents of the great stream of movement called civilization."

To me, the attitude of university professors of mathematics at the present time toward teacher training is most encouraging. Just within the past few years, most of them have come to a realization of the fact that the sort of freshman mathematics students we are to receive five or six years from now depends upon the sort of mathematics teachers we send into the field this year, and next, and next. In my opinion, the reaction of a student, and hence the benefit to the student from a course in mathematics, history, English, or science in high school, or in a university, depends more upon the teacher than upon the subject matter content. Please do not infer from this statement that I consider the personality and professional training of the teacher more important than the subject matter.

It is a comparatively easy task to list the subjects in mathematics with the approximate amount of time to be devoted to each, which should constitute the requirement for a prospective teacher of high school algebra. It is not difficult to list the number of credit hours in educational psychology, history of education, educational measurements, methods of teaching, and practice teaching, necessary for the professional training of that teacher. These

* Paper read at the St. Louis meeting of the National Council of Teachers of Mathematics in February 1940.

are the tangible things that can be tabulated, printed, and talked about with more or less common understanding. Even in the same institution, however, it is not likely that any two students derive the same benefits from a methods course. The results will be still more different in two schools where one course in predominantly a professionalized subject-matter course, and the other is an inspirational course affecting attitudes, appreciations, enthusiasm and interests in the subject and in the young people concerned.

The folder recently prepared by the National Committee on Teacher Examinations, under the auspices of the American Council on Education, interests me because of its implications concerning the relative importance of the various phases of teacher training. The actual questions are not yet available. I assume that Ben Wood will be chiefly responsible for their formulation. They will be of the objective type, with an actual writing time of 11 hours. The assigned time for the examination is as follows:

English Comprehension ..... 40 minutes
English Expression ..... 40 minutes
Reasoning, emphasizing quantitative and non-verbal mental ability. 40 minutes
General culture ..... 180 minutes

1. History and Social studies
2. Current social problems
3. Literature
4. Fine Arts
5. Science
6. Mathematics

(Note that mathematics is one of the six items constituting the examination on General Culture)

Professional information ..... 120 minutes

1. Education and Social policy
2. Child development and educational psychology
3. Guidance and individual and group analysis
4. Secondary school methods

Contemporary affairs ..... 60 minutes

The items just mentioned constitute the divisions of the general examination. In addition to these items, the candidate is to spend 90 minutes on each of two subject matter fields which constitute his major interest.

I do not mention the various divisions of this proposed examination as a guide for the preparation of teachers, but merely as being indicative of the broad training which our high school algebra teachers should have if we are to accept the judgment of the leaders in education who are responsible for this examination.

You will note that the examination of professional information occupies two hours time out of a total of 11 hours. Presumably each of the four topics will occupy approximately 30 minutes. I assume that such topics as The Community and the School, Education and Democracy, and perhaps various courses in the history of education, and in the philosophy of education would contribute to the information and attitude of the student for the first topic. I am convinced that this topic, "Education and Social Policy," does not depend as much upon the text book or references, or even content, as it does upon the experience and vision of the persons who present such courses.

The second topic, "Child Development and Educational Psychology," is more uniformly presented in the departments of Educational Psychology of the various schools of education. For our specific topic of training teachers of algebra, we would naturally turn to Thorndike's *Psychology of Algebra*. I fear, however, that those of us who have done so have been disappointed if we were looking for the principles of psychology applied to the teaching of algebra. There are some helpful statements in this book, such as the following: "Teach nothing merely because of its disciplinary value, but teach everything so as to get what disciplinary value it does have." . . . "The mere knowledge of the language of algebra has more utility than educators have thought, while skill in computing has less." These are typical of many valuable statements in Thorndike's book, but we will have to look elsewhere for a presentation of laws and theories of

Educational Psychology. Judd's *Psychology of High School Subjects* should also be helpful, but I have found no specialized text of this type that can be successfully substituted for a standard, general course in Educational Psychology. I feel quite confident that the teacher of algebra who takes this examination will be seriously handicapped unless he has had such a course.

The third topic, "Guidance and Individual and Group Analysis," should be quite thoroughly covered by the standard courses in guidance and in Educational Measurements. Possibly an intelligent, experienced teacher of algebra "picks up" sufficient skill in these phases of teaching for his work to be acceptable, but with the numerous guidance courses, Educational Measurements courses, and Statistics courses which are now being offered, the wise teacher will make some special preparation in these fields.

The fourth topic, "Secondary School Methods" is simple in its statement, but is really a big order. Even for our algebra teacher, it might mean methods of the senior high school, methods of the junior high school, methods in mathematics, methods in algebra, supervised observation, supervised teaching, cadeting, internship, in-service training, or experimental and research study. Perhaps it is because of my limited experience and knowledge in the first three topics under Professional Training that I am content to leave these subjects as they are now presented in many colleges, with the expressed belief that they are well worthwhile. May I discuss with you more in detail the methods in high school algebra. This includes a theory course with readings, reports, and discussions; supervised observation of the teaching of algebra classes with reports and discussions; and supervised teaching of algebra. From my point of view, all three are absolutely essential. If the college student can establish his concepts and ideals, and means of presentation of algebra through theoretical discussions, then through observing others teach, and finally, by doing the teaching himself, he will enter the field of teaching with definite objectives, and with sufficient confidence in his ability to achieve these aims

If the teachers of courses in University mathematics have not "sold" mathematics to the students who plan to teach, that is the first task of a methods teacher. I notice that the report of the Committee on Training of Teachers of Mathematics, (American Mathematical Monthly, May, 1935) of which Professor Hassler was a member, recommended that the graduate students in mathematics should be supervised in observing and teaching college classes. May I say "amen" to that. It seems to me that one of the unpardonable sins of our present system in college is to assign to a freshman class a highly intelligent graduate student who knows much about mathematics, but very little about boys and girls. We lose many of our prospective teachers of mathematics in their freshman year, when they confront this type of college teacher. The committee just mentioned recommended that the methods courses should be under the direction of professors with graduate training in mathematics, who have taught mathematics on the secondary level, and who have maintained contacts with the secondary field. That is excellent. I suppose I am willing to subscribe to every item in that recommendation because I can qualify, but is it inconsistent to recommend that every teacher of mathematics to freshman classes (in which our prospective algebra teachers will be found) should have had experience in teaching high school mathematics? These freshmen are only three months removed from having been high school pupils, and they most certainly need the most skillful and most experienced teachers that can be found on the mathematics faculty.

If I were responsible for the assignment of classes to the members of a university faculty in mathematics, I would ask the individuals who are represented by the

members of this audience, who are on mathematics faculties, to teach the algebra, trigonometry, and analytics. I would assign the advanced courses to the young Ph.D.'s on the faculty, who have had no experience or training in teaching, but who have recently mastered these advanced courses. It just isn't done. I realize that. But why not?

The simple fact that a university senior "likes" algebra is no assurance that he will succeed in making his students "like" it next year. On the contrary, the very fact that he likes it, is evidence that it is probably easy for him, and for this reason he will not appreciate the difficulties of high school boys and girls. They will soon be asking "what's the stuff good for?" The algebra teacher must be led to appreciate the mathematical nature of the universe. He should know something of the contributions of mathematics to civilization. A course in the history of mathematics is almost indispensable. *Mathematics for the Million*, by Lancelot Hogben, is very helpful if we do not create an aversion toward the book by requiring too detailed reading.

Somewhere during the college course, our prospective teacher should be interested in such facts as the enormous amount of mathematical computations made by Adams and LeVerrier, which led to the discovery of Neptune. Their attention should be called to the marvelous work at the Naval Observatory at Washington, concerning eclipses, tides, chronometer corrections, etc. They should know something of the mathematical deductions of Maxwell, as related to wireless telegraphy. They should know of the work of Pupin in connection with long-distance telephones. They should know to what extent improvements in the flute, organ, and sound-wave photography are dependent upon mathematical contributions. They should know that at the present time text books have been written on the following subjects: Pharmaceutical Mathematics, Mathematics of Investment, Mathematics and Field Artillery, Mathematics for Chemistry Students, Textile Mathematics, Biomathematics. The very fact that these texts have been written indicates that the writers recognize that advanced students in these subjects need a comprehensive review of the mathematics involved. Publishers would not go to the expense of printing such books if they did not feel confident that there would be a demand for them.

About three years ago, I was greatly interested in meeting a young woman who was starting a graduate course in Speech. She had been elected to Phi Beta Kappa in her senior year, but had taken no mathematics throughout her college course. When I met her, she was sitting at a desk pouring over the contents of a text in trigonometry. She explained to me that on the second day in her course on Stage Craft, she had confronted references to radian measure, tangent of an angle, angle of elevation, etc., under the discussion of stage lighting, and the proper angles at which the light should strike the stage settings. She remarked that unfortunately, as an under-graduate, she had not recognized the necessity of having mathematics, and now, as a graduate student in Speech and Dramatics, she was obliged to go back to the courses ordinarily offered in freshman mathematics, in order that she might understand the particular text she was studying on Stage Craft. If we do not call to the attention of our prospective algebra teachers the various uses of mathematics, such as in the actuarian's office of an insurance company, in the various magazine reports in economics, psychometric measurements, in the fields of Biology, and many others, it is quite certain that this teacher will not be able to convince his high school students of the growing necessity for knowing mathematics if they expect to do advanced work in any field of scientific research.

Even though the teacher of algebra succeeds in impressing the members of his class with the importance of algebra in invention and in the advance of civilization,

the high school boy or girl still asks the question, audibly or otherwise, "Yes, but what does it do for me?" Algebra is a powerful, symbolic language. It facilitates orderly, systematic, concise, brief, unambiguous statement of facts. A class in algebra affords an opportunity to teach more actual English than is frequently taught in a regular English class. The pupil must read carefully, and then must express himself concisely. This is one of the first objectives in English courses, but unfortunately, in many cases, the objective is not realized.

In an algebra class, the student has something definite in mind to be expressed, whereas, in a class in English Composition, he spends about two-thirds of his time and energy in attempting to discover or invent something that is worth saying, ignoring the objective of desirable brevity and concise statement by endeavoring to expand this one meager thought into the required 500-word theme. A statement in Everett's *Fundamental Skills of Algebra* has always impressed me: "The child needs algebra for what it will enable him to do, but infinitely more does he need it for the orderly way in which, in the midst of constant change, it will enable him to think." Incidentally, may I suggest that every prospective teacher of algebra should be required to read this book by Dr. John P. Everett.

The teacher of algebra should be led to see the opportunity to teach the importance of neatness and the disastrous results of inaccuracy. I know of no other subject that lends itself so readily to this objective. A misspelled word in an editorial does not seriously interfere with the accomplishment of the purpose of that editorial. The wrong pronunciation of a word in a sermon is not likely to decrease the number of converts. An error in grammar in the impassioned final plea of a lawyer to the jury will not diminish his chances of bringing that jury to his point of view. An error on the part of the journalist, the preacher, or the lawyer is not fatal. But if the engineer makes the insignificant mistake of misplacing a decimal point in computing stresses and strains in a plan for a railway bridge, the crash of that bridge is a calamity.

We dare not state in the presence of a psychologist that algebra helps to establish the habit of concentration, but I do not hesitate to say that the solution of a problem in algebra enables me, by means of the moving pencil point as I proceed with the steps, to focus my attention on the work more completely than I can in reading a text in History, or Geography, or Literature, and it leads me to appreciate the value of the power of concentration.

If our beginning teacher believes in algebra, and appreciates its possibilities, he is over the first hurdle. It might be well to read Dewey's monograph, "Interest and Effort in Education," in order to be convinced that interest is a necessary, if not a sufficient condition for success. Interest produces effort—effort increases interest, and a combination of the two usually results in success.

Methods of study, teaching pupils how to study, helping pupils to budget their time by means of a study schedule, a supervised study period, the assignment of the lesson, an assignment that will facilitate and give reasonable assurance of the desired preparation—all of these and many more are the responsibilities of the teacher. They are, therefore, the responsibility of the college professor in training this teacher.

Mathematics teachers are more stable than the so-called educators. The mathematicians are the stand-patters, and educators are the faddists. Of course, the educators would say they are more progressive, and that the mathematicians are more conservative. I fear I am a conservative-progressive. I have never seen the necessity of spending much time in education courses discussing the socialized recitation, the project method, the unit method, the contract method, and all the other fads that come and go. I admit that there

is something of value that is salvaged from every new hobby, such as the integrated curriculum, and the integrated child, but no one of these is going to provide for the salvation of public education. May I illustrate what I mean by being skeptical about what I have dubbed "fads." In about 1925, or '26, Nebraska educators were shouting from the house tops concerning the project method. Students from other classes implored me to help them plan projects in high school mathematics. I told them I didn't know how, and asked them to work out their own projects. In the fall of 1927 I registered in Teachers College, Columbia University, and immediately registered for a course under Dr. Kilpatrick, who had been one of the chief advocates of the project method. I welcomed the opportunity to learn all about this much lauded Project Method. I sat in his class from September until June. He did not mention the word "project." That was a thing of the past, and at that time he was talking about "indoctrination." The project method by this time has been superseded by other fads in practically all educational institutions.

The ability to make a good examination that does not put a premium on memorizing, and does not over-emphasize the manipulative skills, as Everett aptly calls them, and at the same time is so constructed that it can be accurately and uniformly scored, has been a challenge for many more years than the fads just mentioned, and will continue to demand our attention after they have been forgotten. Just what is a good examination? What is its purpose? Is it a testing or a teaching procedure? Evidence of progress is probably the best means of motivation in any subject. Hence, the many "timed tests" that are found in the newer texts. These are defensible on the grounds that they do measure progress, and therefore maintain interest. But beware of the dangerous emphasis upon mechanical efficiency rather than improvement in the manner of thinking.

Many schools are making statistical investigations in school population, taxes, grade placement, correlation between intelligence and school success, etc. Who is better qualified as a consultant in this work than the algebra teacher? This means a course in statistics—perhaps mathematical statistics, deriving and justifying statistical formulas, perhaps educational statistics, in which we accept formulas without proof, and then illustrate their use. I teach Educational Statistics. We compromise by having many of our students take both courses.

Whenever I step into the office of a lawyer or a physician, I am impressed by the appearance of his library, and the number of professional magazines on his desk. The professional library of our teacher is woefully neglected. It is our duty to provide our students with a bibliography more elaborate than a list of required readings. References to THE MATHEMATICS TEACHER, *School Science and Mathematics*, *The National Mathematics Magazine*, and the *American Mathematical Monthly* should not be so frequent that our students feel relieved to escape from these required readings, but they should be led to appreciate the necessity of subscribing or having the school subscribe for one or more of them.

What about a Mathematics Club? Is it a desirable asset in creating and maintaining interest in mathematics? Most of us believe that it is. Do we dare to hope that our algebra teacher can organize and supervise such a club when he accepts his first school, unless he has participated in the activities of such a club? For many years, in addition to a chapter of Pi Mu Epsilon, we had a Secondary Education Club, in which the mathematics students, the English students, the language students, students in the social studies, and those in the sciences took turns in presenting programs. It was enjoyable work, and proved to be exceedingly valuable to our students. Unfortunately, it has been abandoned. No one knows why. Possibly the

emphasis on research that seems to be necessary in order to make a university professor respectable, has contributed as one cause for our failure to find a sponsor for this club. The chapter of Pi Mu Epsilon is still active, but it does not take the place of the Secondary Education Club.

The algebra teacher is responsible for recommending texts and equipment for the recitation room, and perhaps the arrangement and kind of seats. The seating of a room in which pupils are frequently sent to the board should be such that any individual can go to the board or return to his seat without stumbling over the feet of his neighbors. In other words, the rows should go from front to back, with aisles between, rather than across the room from one side to the other—the usual arrangement in many class rooms. The bodily comfort of the pupils—heating, ventilation, and lighting are among other responsibilities of the teacher.

I have purposely refrained from discussing what we might consider professionalized subject matter. It is probably more essential in the methods of junior high school than for the senior high school, since much of the content of the junior high school course has been met only incidentally by our present university students, and not as a part of any previously required mathematics course. When and how shall we teach directed numbers? When, how, and to what extent shall we present graphic work? What about the formula? Shall we teach the clearing of fractional equations by changing each term to the common denominator and then dropping the denominator, or shall we multiply both sides by the least common multiple, and never allow the pupil to see the equation written with the common denominator? How important is the function concept? How shall we introduce logarithms? Should we explain that $2\frac{3}{5}$ means $2+\frac{3}{5}$, but if we replace the "2" by "b" it means $b\times\frac{3}{5}$? Yet algebra is generalized arithmetic. Should more, or less, emphasis be given to verbal problems? These, and a thousand other questions might be discussed. I have an answer to each, but I cannot justify my answer in opposition to yours. No two of your students will teach just as you do and it is well. No two of mine will follow in my footsteps, and that is fortunate.

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What Specialized Knowledge Should the Teacher Training Program Provide in the Field of Geometry?

By P. D. Edwards
Ball State Teachers College, Muncie, Indiana

The answer to the question above is of course dependent on the amount of time allowed for the study of geometry. I shall discuss the courses which we offer at Ball State Teachers College and the objectives in each course.

At present in Indiana the teacher of Mathematics is required to include a total of 24 semester-hours in mathematics in his preparation. The distribution of these hours is specified. A definite amount must be taken in algebra, trigonometry, calculus, mathematics of finance, geometry, and methods. The requirement in geometry is six semester hours. Ball State operates on the term basis and our courses meet four hours per week, thus being equivalent to $2\frac{1}{2}$ semester hours. To meet the requirement our graduates must take two geometry courses in addition to the Freshman course in Analytic Geometry. We are now offering five courses from which the student may choose two. These five courses are (1) Second course in Analytic Geometry, (2) College Geometry, (3) Projective Geometry, (4) Non-Euclidean Geometry, (5) Field Measurements.

The first of these is a continuation of the first course in Analytics and completes with the first course slightly more than is covered in the usual five semester hour course in the Universities. As the content of such courses is familiar to most of you I shall not take time to discuss it other than to say that students who expect to do graduate work in either the field of Mathematics or in the field of Physics are urged to choose it as one of their courses.

We have been offering the course in College Geometry for about twelve years. When we began the only available American text was the well-known book by Professor Altshiller-Court. Later there appeared the Modern Geometry by Professor Johnson of Hunter College and last year my colleague Professor Shively published a text entitled *An Introduction to Modern Geometry* which is finding considerable favor among teachers. Three English texts should be mentioned in this connection. All bear the title *Modern Geometry*, and were written by Lachlan, by Durell, and by Godfrey and Siddons.

I am not representing any book company nor any author on this program, so I shall not make any comparison of texts. The objectives which I shall outline can be met using any of the books as a basic text. The first objective I have in mind when giving the course is to extend the student's knowledge of the field of elementary geometry: that part which is most closely related to the material taught in high school. You who heard Professor Wren's talk at the Columbus meeting of the Council saw how the usual angle side angle construction could be related to more than thirty additional construction problems by using relations involving an altitude and the radius of the circumscribed circle. By adding the medians, the angle bisectors, the radii of the inscribed and circumscribed circles, the perimeter, sums or differences of two sides, or of two angles the number of possible constructions grows amazingly. The usual high school text contains five theorems on congruent triangles. By including the elements named and a few others, more than five thousand theorems and the corresponding construction problems are possible. Of course I have never

attempted to include them all, but every student taking a course in Modern Geometry should be able to construct and prove any one of this large number of problems. In my courses in the training school I have found that high school students also can readily grasp the basic notions and that they appreciate the broader viewpoint. The better students frequently do as well as the average college student. While this part of the work is primarily concerned with the Geometry of the Triangle, the student develops, almost as much, his ability to work with other parts of elementary geometry, particularly the work in circles and with parallels.

The second objective is to broaden the student's horizon by the introduction of many new fields. To avoid superficial thinking I have found it very profitable to require one or two term reports. The student chooses or is assigned a topic and is allowed about three weeks in which to prepare a paper. Original work is encouraged and in many cases results have been obtained which are original so far as the student is concerned. A few of the papers which I have filed away in my office contain as much original work as is to be found in the average M.A. thesis. Most of the results are doubtless known to others, but are not to be found in the books and periodicals of our own library and so are an excellent introduction to graduate study. While this intensive work is being prepared the class work goes on and a dozen or more topics are taken up. One result is always evident. Each student acquires a viewpoint. One may be writing a paper on Inversion. Soon I find him using the method of inversion to prove theorems in other fields and indeed by inverting the plane discovering new theorems on circles from known theorems on lines, or conversely. Another may be working on homothetic figures or on co-axal circles and his approach to problems likewise shows the result of his special point of view.

As there are topics which are treated in both Modern Geometry and in Projective Geometry I have usually followed the plan of assigning such topics to students who have not studied Projective Geometry. This applies particularly to such topics as Cross Ratio and the Principle of Duality. In a few cases I have assigned such subjects to one who has studied Projective Geometry, but in such cases I have demanded and obtained much more advanced papers than would otherwise be required.

To summarize the objectives of Modern Geometry, I put first, a development of a *high standard of proficiency* in work closely related to the field of elementary geometry; and second, a broad knowledge of many special topics that have developed in modern times with intensive work in two or more fields. As part of the final examination I require a student to outline the principal results, theorems, etc., in one or more fields other than those in which he has written his term papers. This requirement is always announced early in the term and serves to do what out friends in the Department of Education call "motivate the work."

While we offer five advanced courses, we do not offer all in any one year. The result is that not many have the opportunity to take Modern Geometry and Projective Geometry. The subject matter can be so chosen that there is not much duplication, but there is an overlapping of objectives. The basic notions of central projection and section, of cross ratio, and of duality are common in both fields. In projective Geometry they become the central ideas of the course. For students who go into graduate work, perhaps a course in Projective Geometry is the more valuable. For those who will spend their lives teaching in the secondary school the advantage is perhaps on the side of Modern Geometry. Projective Geometry has for years been a recognized part of the curriculum of the graduate schools and many texts are much too advanced for the average junior and senior in our teachers colleges, but there are *a few*, such as the ones by Dowling of Wis-

consin and Holgate of Northwestern that are elementary enough for undergraduates. Much that has been said about the second objective of Modern Geometry applies as well to Projective Geometry. Both decidedly alter the student's conception of the limitations of the field and serve to impress him with the magnitude of the subject.

Non-Euclidean Geometry is generally conspicuous by its absence in American undergraduate instruction. It is my opinion that this is exceedingly unfortunate. The lack of suitable texts is no doubt one reason. Most of the texts in the English language are written for advanced students. For a while I used as a text *Non-Euclidean Geometry and Trigonometry* by H. S. Carslaw of Sydney, Australia, published by Longmans, Green and Co., of London. Unfortunately it is now out of print. I bought all the used copies I could find and now give the course mostly by the lecture method.

I have often said that this course is at the same time the most valuable and the least valuable course that I offer. It is the least valuable in that it is the most removed from the subject matter field of elementary mathematics. It is the most valuable in that *it* more than any other course typifies the true nature of all mathematical reasoning. In all mathematical work we draw necessary conclusions from a set of postulates. Nowhere else is one forced to rely on pure logic to so great an extent. The Euclidean postulate of parallels is not required by the other postulates of Geometry nor is it required by the physical nature of the world. In some branches of mathematical Physics it is not even the most convenient postulate. All of you who have taught ninth grade algebra have found students who had trouble with the man who rowed a boat across a river. The man rows three M. P. H. in still water and the *current* flows at the rate of four M. P. H. You know the story. Most of us have been inclined to regard the student as dumb who couldn't find the resultant motion. The same situation arises repeatedly in Trigonometry, Calculus, and Physics whenever one attempts the composition and resolution of vectors. Yet after all, these rest on nothing more secure than postulates. There is undisputable evidence that velocities which approach the speed of light cannot be combined according to the rules of Euclidean geometry. The true result can readily be computed by the Trigonometric formula of the triangle in the Hyperbolic plane of Lobatshewski and Bolyai. Likewise certain other results can best be interpreted in terms of the Elliptic Geometry of Riemann. I do not present these as practical applications of the non-Euclidean geometries, but certainly the teacher of Geometry needs to appreciate the nature of the course which he is giving. Euclid's Elements have proved useful to the human race for more than two thousand years. There is no doubt that the human race will still be using the same basic material for the next two thousand years. We will build our bridges and skyscrapers by formulas based on the Elements. We probably will *not* find Euclid adequate for our scientific refinements. Aside from its possible applications, the discovery of non-Euclidean geometries has had a tremendous effect on the development of many branches of science. No longer may it be assumed that consistency alone is a valid test of the assumptions on which a logical structure is erected. Any consistent system of postulates may give rise to a logical structure which is exactly as true as the postulates. Consider for example the Phlogiston theory of the ancient Greeks. Take time out some day and study it. Consider it only on the basis of its logical structure. It was a fitting companion for Euclid's Elements. Both rested on Assumptions, neither was required to stand the test of today's scientific investigations. The phlogiston theory was discarded more than a century ago when it proved insufficient for the needs of the founders of modern Chemistry. Yet today when the heating engineer talks of the number of B. T. U.'s in a ton of

coal or in a gallon of oil his remarks could be explained on the basis of the phlogiston theory quite as well as on a theory based on the Bohr atom. Again the geocentric theory of the solar system in the hands of Ptolemy was a beautiful piece of logic. It had the additional advantage of adding immeasurably to man's idea of his own importance. It stood for more than a thousand years and even today we speak of sunrise and sunset. These terms can be taken literally under the Ptolemaic theory, but become mere figures of speech under the heliocentric theory of Copernicus. While speaking of Ptolemy may I suggest that you look up the theorem known by his name, namely, "The product of the diagonals of an inscribed quadrilateral is equal to the sum of the products of the opposite sides." You will find it on p. 26 of Halma's translation "Composition Mathematique de Claudius Ptolemy." (Stechert) It is a simple theorem easily proved by a high school student by means of similar triangles, yet in the hands of an expert it became the principle theorem by which the first set of trigonometric tables was constructed. Incidentally, it makes a good topic for a mathematics club paper.

Modern Physics furnishes us with many examples of perfectly logical theories that have been superseded by new theories based on new hypotheses. The disturbing feature is that the newer hypotheses are always less "evident" than the earlier. The working rule appears to be to choose the simplest theory that will explain all known facts. At the same time the older theory is kept for many of its practical applications. So today the atom is indivisible in ordinary chemical reactions and at the same time we explain those same reactions by having an exchange of electrons between atoms in polar compounds and a sharing of electrons in non-polar compounds. Perhaps in the not distant future we will study the non-Euclidean geometries as tool subjects for scientists while keeping the Euclidean geometry for the use of engineers. Certainly today the mathematician can afford to give some time to their study. I feel quite sure that no subject which I have studied has affected my own thinking so much as the course in non-Euclidean geometry. All of this is intended not as a plea for non-Euclidean geometry as a tool subject, but rather as a splendid exercise in abstract thinking.

The last course I wish to discuss is "Field Measurements." It is listed in our catalogue under the title of "Mathematics of the Junior High School." Neither title is completely descriptive. The course was introduced in response to a felt need and has been accepted by the State Department of Public Instruction as meeting in part the requirement for courses in Geometry. It is in part a course in surveying. Students are taught the use of the level and transit. It is in part a course in Astronomy or Geography. Students are taught to find the latitude and longitude of Muncie by means of the sextant and also by the use of shadows and by the hypsometer. In part it is a study of junior high school mathematics, as students are required to review at least three series of junior high school texts. In part the course is different from all these things in that one of the requirements is the review of several foreign texts. I had the library purchase the entire list of foreign books given by Professor Cairns in the American Mathematical Monthly of Jan. 1935. This may sound to you like a hodge podge. Perhaps it is; I have given the course about eight times and have never yet done exactly the same things twice. The inspiration for the course came from the little book "Field Mathematics" by C. A. Schuster and F. L. Bedford. I still have the students purchase a copy and strangely there are seldom any second hand copies for sale. Students wish to keep the book for their own use when they begin teaching. In few courses have I ever been more conscious of the objectives and more uncertain of my accomplishments. Most of our graduates begin their teaching careers in junior high schools. This is due to the fact that most of our Indiana schools

have the 6–3–3 form of organization and to the additional fact that most of the employing officials seem to prefer experienced teachers in grades 10–12.

When a student's last course in college mathematics is in Calculus or perhaps in the Mathematics of Finance and his first teaching experience is in grade 7 a considerable adjustment is necessary. Our primary objective in putting this course into our curriculum was to aid the student in making this adjustment. Perhaps it should be classed as methods rather than as mathematics. However the emphasis is placed on geometric material and so I am including it in this discussion.

The examination of foreign texts is for the purpose of acquainting students with the fact that the United States has lagged in the matter of reorganization of mathematics. This movement was begun about the beginning of the present century and received the endorsement of the International Congress of Mathematicians. In some countries because of federal control the change came quickly. In this country the reorganization has been advocated by many of our leading mathematicians and by a number of leaders and would-be leaders in the field of education. It has been rather strenuously opposed by most teachers of secondary mathematics and by many mathematicians in the colleges. The majority of both groups are perhaps unacquainted with the advantages that might follow and are apprehensive of the disadvantages that might follow. I do not regard our system as bad but on the other hand it is not perfect. My only purpose is to point out to my students that other organizations of material are possible and that certain advantages may follow some of the changes.

The examination of current series of junior high school texts is for the purpose of acquainting students with the type of teaching that will be expected of them in the junior high school. Emphasis is placed on geometric material. Intuitive geometry may be taught by practical measurements. Work with the plane table receives more attention than the use of transit and level although these instruments are also used. A home made plane table and hypsometer cost little and are so simple in construction that the emphasis in the mathematics class may be placed on the mathematical principles involved. In the junior high this is important. The aim is to teach mathematical ideas rather than the use of an instrument of precision. Because of the cost of a transit the pupil usually cannot be allowed to handle it but must be a spectator. As a learning experience this is far less valuable than active participation with a simple instrument.

In conclusion I wish to point out the three separate phases that are present in the development of a science, and the contributions of these courses to the preparation of a mathematics teacher when viewed in this light.

The first phase in the development of any science is systematic observation, accompanied perhaps by experimentation. Following closely is classification of results and the formation of postulates concerning generalizations.

The second phase consists of the logical deduction of necessary consequences which follow from the postulates.

The third phase consists in predicting results that may be checked by further experimentation.

The bulk of the work of the undergraduate student in the biological and social sciences is in the first phase. Most of our work in college mathematics is in the second phase. The engineer and the applied scientist is constantly working in the third phase.

In the lifetime of the mathematican he passes through each of the stages not alone in geometry but even in the elementary number work. The most important facts of Geometry and Number are learned by the intuitive and experimental methods. We must not lose sight of this fact in the preparation of teachers. It is because of this fact that the course in field measure-

ments was added. We believe that it is worth while and that it is helping to serve the purpose for which it is intended. The teacher in the junior high school is dealing with students whose maturity is such that they are not ready for the second phase of geometry.

In the other courses which I have discussed the training is directed toward the logical reasoning. That of course is the central part of all mathematical thinking.

The third phase takes us to the application of mathematics. The field work course does this in a small measure. Ultimately of course this phase of development falls to those fields in which mathematics is a tool. That however is not included in the topic which I set out to discuss.

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Training Teachers of High School Arithmetic

By JUDSON W. FOUST
Central State Teachers College, Mt. Pleasant, Michigan

ONE is confronted with the necessity of choosing in a very limited way when attempting to discuss in a few minutes the training of teachers of high school arithmetic and keep in mind as a background theme the student who is not going to college. May we then assume the necessity of a broad liberal education, sound scholarship, professional training and a period of supervised teaching. This leaves us the time to bring into sharp relief a few points whose emphasis seems most likely to produce worthwhile improvement in this area of teacher training. In doing so it may be necessary in giving direction to our teacher training program to discuss somewhat in detail the aims and methods and points of view to be employed in arithmetic itself.

While it may seem rather general, the first suggestion to be offered for the preparation of teachers of arithmetic is that the importance and the dignity of arithmetic must be emphasized. I can think of no weaker link in a teacher training program than to send ou tindividuals unaware of or indifferent to the true place of that subject in the educational pattern and its importance to the individual and society as a whole. I can think of no greater initial advantage, pointing to the success of a teacher and his subject and the ultimate growth of the individual taking it, than a real belief in the importance and ultimate contribution of that subject—if that belief is the outgrowth of adequate training and experience within that field.

It is not my intention to consider arithmetic as a substitute for any traditional high school mathematics course nor to consider it merely for a limited group of students such as the non-college-preparatory group. Neither is it to be considered only for poor scholarship risks, slow learners or misfits. If properly taught by properly trained teachers it should have high value for all students whether going to college or not, whether following a general or a vocational course, whether of high or low scholarship.

One might ask why this is not the prevailing opinion or at least not the prevailing practice. Also why is it that at the higher levels we are afraid of the word arithmetic, as if it could not stand by itself, and substitute the title *Commercial Arithmetic* or *Business Arithmetic?* Why do we disguise it under the more profound title *Business Mathematics* or the attractive titles of *Mathematics for Life* or *Mathematics at Work?* An exception is found in a book which courageously calls itself *A Higher Arithmetic* but the general trend is apparent.

Was this not brought about partly because arithmetic, as such, had become rather barren for the average student and we sought the use of the vocational aim for motivation? Was it not partly because arithmetic was conceived and taught largely as a tool subject, a drill subject, and a skill subject rather than a *truly mathematical subject* in which concepts and meanings and relationships and thinking were stressed?

Perhaps as such it was well that arithmetic should lose face. But as we think of preparing teachers for a *new arithmetic* we must think of preparing for a subject that must make its contribution to a developing student through concepts and meaning and understanding—through developing his powers to recognize and analyze and interpret and represent relationships found in a multitude of varied situations. As such, arithmetic must assume a role of importance and a dignity not now fully recognized. In the words of Buckingham before the 1937 meeting of the Council,

"When arithmetic is taught with meaning it ceases to be a bag of tricks and becomes, as it should be, a recognized branch of mathematics." Unless we send out teachers who are thoroughly aware of the purpose, the importance, and the opportunity of arithmetic and who have the training and outlook that will make this subject reflect the true spirit of *mathematics*, with all that the word implies, it is of little worth to discuss details of the training.

The implication for the teacher training program is apparent. The prospective teacher must gain an understanding of the true spirit of mathematics. This, it is suggested, can be secured only by experiencing the discipline of mathematics by pursuing that field of study. Morton, at the joint meeting of the National Council of Teachers of Mathematics and the Mathematical Association of America in 1938, commented in detail with respect to the necessity of a mathematical background. In part, he said, "I could continue at some length to recite to you the difficulties which teachers of arithmetic experience because they lack a subject matter background to reinforce them as they try to understand and then to teach certain topics in this subject. The need for additional experience in mathematics becomes so obvious to one who works with teachers and prospective teachers that it is very difficult to understand why the matter should be debated at all." This would seem to oppose the practice often used of assigning a class in arithmetic to almost any one in the high school who needs another class to complete the teaching load. Such persons are not qualified; neither is the average person who has taken only the traditional mathematics courses. Morton suggests for elementary teachers the minimum experience in mathematics of college grade in a one year course of from 6 to 10 semester hours organized especially for this purpose. For teachers of arithmetic in high school we certainly should expect this together with additional work of college level such as college algebra, trigonometry, statistics and mathematics of finance or investment.

In fact, since arithmetic in the high school will generally be taught by the teacher of other mathematics this requirement of pure mathematics will be met but the special course to cover professionalized subject matter of arithmetic may be lacking. In any case the student soon to become a teacher, must somewhere become aware through experience that mathematics is concerned with relationships—quantitative, spatial, and logical. He must see that Arithmetic is more than computation and application. He must begin to see that relational thinking is the central theme and must in turn make the student he is to teach see it.

I have much more faith in the effectiveness of a teacher training program that develops this insight and outlook for the teacher through a study of mathematics rather than by merely discussing it and telling him it is so. Perhaps we should enable him to develop it through broad sound mathematical training and then discuss it. At least to merely discuss it will not prove enough.

It is always a source of stimulus to me as a teacher to see the lack of concepts and understanding and grasp of quantitative relations in students preparing to teach arithmetic in the elementary grades. I begin to think of the consequences if these people go out to teach arithmetic without considerable improvement. Likewise it is a source of stimulus and satisfaction to see the eagerness with which the great majority of them enter into the program of improvement when the purpose of mathematics is made a little more clear, when the processes begin to be understood and when they begin to feel their ability in this area grow.

A young lady who was taking geography came to me a short time ago and asked if I knew the formula for the circumference of a circle. I replied that I did and stated it. She looked at me a moment and asked, "Are you sure it isn't $C = \pi r^2$?" I assured her that I was very sure it was not $C = \pi r^2$

and proceeded to show her that length would not be represented by a formula involving the product of two lengths as r^2. I carried the discussion further and illustrated with various formulas for length, area, and volume until she could pick out what each of the formulas stood for even though she did not know to what geometric figure it applied. She left in a state of enthusiasm not only thanking me for answering her question but commenting that she wished she had been taught some things like that in arithmetic.

Any alert teacher could furnish example after example of a similar nature showing the surprise and satisfaction the student gets from insight into something with which he is more or less familiar but has never understood. Every year it repeats in training teachers of arithmetic when the number system is studied—how it is built and how it operates, when division is explained as continued subtraction, when percentage is shown to be nothing but problems in fractions stated in a new language—and so on indefinitely. It all sounds so simple and it is. We must provide the background of mathematics. First it must provide for insight into mathematics and its methods and purpose and spirit. In the second place it must provide professionalized subject matter courses in which the field of arithmetic itself is covered slowly, thoroughly, intensively, extensively so as to lay again the basic concepts and prepare an understanding of the processes, techniques and meaning of arithmetic as a mathematical subject.

In emphasizing the need to dignify the work of arithmetic by recognizing it as a truly mathematical subject, and by emphasizing concepts and meanings and relational thinking there has been no intention to ignore nor minimize the social and economic applications of arithmetic. Their place is to be retained but the viewpoint and approach and treatment of them may be altered. They are viewed as the *material in which* the student learns to recognize and abstract and analyze quantitative relationships as they occur in varied problems.

The teacher training program must clarify for the prospective teacher of arithmetic the distinction and relation between the mathematical and social phases of arithmetic and the purpose of each. It must do this by outlining in some detail the whole discussion, but also by taking the student through a considerable portion of arithmetic much as he, when a teacher, will find it necessary to direct a group of students. A number of our goals in training teachers of arithmetic will thus be the goals of arithmetic itself although perhaps on a higher level.

Perhaps no clearer statement of the place and importance of the social phase of arithmetic, and its relation to the mathematical phase, can be found than in the report of the National Council Committee on Arithmetic appearing in THE MATHEMATICS TEACHER for October 1938. The report says in part:

"The functions of instruction in arithmetic, then, are to teach the nature and uses of the number system in the affairs of daily life and to help the learner to utilize quantitative procedures effectively in the achievement of his purposes and those of the social order of which he is a part. This conception recognizes two major mutually related and interdependent phases, namely, the *mathematical* and the *social*. Both phases are essential. Neither by itself embodies appropriate instruction and neither will develop in the minds of the pupils the arithmetic that they should learn. Mere grasp of the mathematical phase does not provide for complete social significance, and, conversely, a too exclusive emphasis upon the social phase will not guarantee grasp of the mathematical.

"The purposes of stressing the mathematical phases are:

1. To develop the ability to recognize among all the attractive and objective elements of appropriate situations the number element that does not meet the eye, but nevertheless, gives them their order and exactness;

2. To introduce pupils to the systematic methods of attack upon the number ele-

ments of situations that have finally been brought together into the unified number system as we know it and use it today;

3. To develop in pupils both confidence in the reliance one can place upon such methods of attack and facility in their use by guiding practice in number thinking to higher and higher levels of effectiveness; and

4. To train pupils to carry to later and more advanced studies of important personal business, social and civic situations those methods of attack with the deliberate purpose of analyzing out their number elements and determining their relations to the other elements which with the number elements comprise the whole.

"The purposes of stressing the social phases are:

1. To insure understanding of the contributions number has made to the development of the social institution that have made possible the progress of the human race.

2. To insure that the work of pupils in arithmetical processes will have meaning and significance; and

3. To develop in pupils the disposition and ability to apply increasingly mature procedures in the quantitative situations of life."

Again under selection of content the committee states:

"(a) arithmetic is an important means of interpreting children's and adult's quantitative experiences and solving their quantitative problems. Consequently, the *content* should be determined largely on the basis of its social usefulness and should consist of those concepts and number relationships which may be effectively used.

(b) the committee believes that the social-survey techniques commonly employed to determine the content of arithmetic are not adequate. This procedure merely selects the arithmetic that *is* used, but gives no indication of what arithmetic *might be* used to advantage. Likewise, the analysis of the processes of arithmetic into component elements for teaching purposes fails to provide the meanings and relationships which should be the core of learning.

(c) Mathematical processes have a high degree of permanence and coherence, but in their social applications certain topics may deserve more emphasis in one generation than in another. The basic mathematical concepts and processes remain the same from one generation to another, but the application and utility of them may vary."

Again under methods of teaching the committee reports:

"To be most valuable socially, arithmetic must be taught with the aim of developing resourcefulness in dealing with quantitative situations. This will not be achieved from mechanistic learning nor from emphasis upon the social values of arithmetic. Resourcefulness may be achieved by pupils who constantly utilize meaning and understanding in their dealing with numbers. . . . Regardless of which methods of teaching and learning are employed in the encountering, recognizing and learning of arithmetical materials the teacher should be conscious of and able to recognize desirable arithmetic concepts, principles and processes in social and economic situations."

If I interpret the stand of the committee correctly there is no necessary conflict between the social and mathematical considerations of arithmetic. Both are needed. The social interpretation determines the content; the mathematical phase determines the method of approach and the method of treatment. To give the prospective teacher the ability to select and to understand such social and economic material and to "be conscious of and able to recognize desirable arithmetic concepts, principles and processes in social and economic situations" certainly calls for a period of work in a professionalized subject matter course in arithmetic. It is a common experience of even good students who have had considerable work in higher mathematics to find that they are quite unprepared in background material if assigned to do practice teaching in arithmetic. An adequate teacher training program must provide them with considerable work in this area.

Lastly, I would not have you think that the place of computation and skills in arithmetic can be overlooked. They are considered last because they have been emphasized most in the past. They are the

tools which sometimes have become the whole of arithmetic. However their place is being understood. For teacher training, as for the student in the high school, 100 per cent mastery is the only acceptable norm. At the college level as elsewhere wide variation of mastery will be found. It means that attention must be given to the fundamental processes with integers and fractions in preparing teachers of arithmetic but the work must not be merely drill work nor merely a blanket review. Here as at the high school level such a procedure is ineffective. The emphasis must be made on understanding the process and through drill in connection with varied applications. It must be diagnostic and individualized. It must be specific and objective. Time and interest are too precious to lose through any general drill procedure. Only individualized techniques prove efficient in the skill area at the teacher training level.

The suggestions for the training of teachers of arithmetic include:

1. Mastery of the fundamental operations through understanding and through individualized, diagnostic procedures.

2. An understanding of the place of the social and economic side of arithmetic together with a background of information and experience through professionalized subject matter courses that will enable the prospective teacher to recognize and make effective use of desirable arithmetic concepts, principles and processes in social and economic situations.

3. Sufficient training in pure mathematics to develop arithmetic effectively, to teach mathematical techniques and processes with insight and understanding, to emphasize the thinking side of arithmtic, the relational aspect and the true mathematical spirit of the subject.

4. To develop through this training a broad understanding and deep conviction of the importance and dignity of arithmetic as a valuable subject of instruction in its own right for every student in the high school.

THE ALGEBRA OF OMAR KHAYYAM

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The author of the *Rubaiyat* was also an astronomer and mathematician. This work presents for the first time in English a translation of his algebra. In the introduction, Mr. Kasir traces the influence of earlier Greek and Arab achievements in mathematics upon the algebra of Omar Khayyam and in turn the influence of his work upon mathematics in Persia in the Middle Ages. The translation is divided into chapters, and each section is followed by bibliographical and explanatory notes.

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Mathematics in the Training of Industrial Workers

By CLARENCE J. LEONARD
Southeastern High School, Detroit, Michigan

LET ME at the start make a distinction between shop mathematics, and technical mathematics or mathematics in the education of industrial men. Shop mathematics usually confines itself to the production of answers to specific problems selected from one or two specialized trades. Technical mathematics covers the basic applications from the sciences of production, including the shops, the drafting rooms, machine design, and the allied sciences such as aerodynamics, principles of electricity, etc. It is this broader field of activity that I have in mind as I present this paper.

The changing conditions in industry require the teacher of technical mathematics to be a virtual prophet. Today we are teaching the students what we presume they will need tomorrow, and all we know about it is what we learned yesterday. But isn't it science and mathematics that give us our only instrument for peering into the future? In comparison history is a mere newspaper account of what man has done with the laws and forces of nature. When we consider that mathematics is the nerve center of all science and industry we have an opportunity in technical mathematics that is not matched in any other place in the curriculum. I do hope that some of the practices we have followed will be of interest to this group.

The changes in industry keep us free from the feeling that there is nothing new for the teacher of mathematics. One of the new elements that has probably the greatest implications for mathematics teachers is the widespread use of precision measurement. The high school boy can measure to thousandths of an inch with a micrometer, the toolmaker measures to a ten-thousandth of an inch with his micrometer equipped with a vernier. The inspector with a dial gage can check dimensions to thousandths or tenths of thousandths as rapidly as he can read off numbers. Johannson gage blocks are used by all the larger industries for measuring to hundred thousandths or millionths of an inch. What does this use of precision mean to the teachers of mathematics? For one thing students must be taught rounding off decimals to a specified figure. They must know some thing of required accuracy in the data, significant figures in the results, and the required exactness through the computation.

Above all other suggestions for technical mathematics for non college preparatory students I advoeate much greater stress on absolute accuracy of computation. In industry as well as commerce 100 per cent accuracy is required. Why not considerable emphasis on it in high school, if we want to make our courses conform to life situations? Accuracy is hard to get, I think perhaps the hardest of anything I ever tried to get. But if you demand it you get it. Perhaps a few words on how we have drilled students for accuracy will be of interest. 100 per cent is the passing mark, and all units except the last of the term must be passed to pass the course. The tests are shortened and confined to fundamentals, but of course they don't all pass on the first trial. They are given three trials in class and after that start coming extra periods. A few have taken tests as many as fifteen times before passing. It is doubtful whether they achieve much of a working knowledge but they seem to appreciate the practical aspect of of the 100 per cent passing mark. We have surprisingly little difficulty in keeping them after school for these tests and most of them are quite willing to admit they are better than they used to be. If they have

been made conscious of what it means to get five of six problems strictly correct that in itself is somewhat worth while. The extra tests can at least be charged to additional drill. We guarantee an "A" grade to all students passing all tests the first time, that proves a big help in keeping the better students strictly on their guard. Let me remind you that the 100 per cent passing mark is no easy way out for the teacher.

Training in accuracy has another point of high value for the student of lower ability. The training in copying down numbers correctly is of more value to them than some topics of algebra they can't understand, and never could use. Modern industry and commerce as well, have many occasions for copying and checking numbers correctly. All stock is numbered, parts are numbered, blue prints are numbered, departments are numbered, to say nothing of quantities of this and that to assemble a given number of units. This may seem like a low level of accomplishment for high school students. The point is we have them with us, and the question is what is the most efficient thing we can do for them?

Another unit of value open to, and within range of students of lower ability, and one which it seems to me does not receive sufficient emphasis in our schools now is neatness and orderliness. No one would have any faith in a sloppy or poorly arranged report.

The interdepartmental reports within a large industry bring up another point within the reach of students of the lower levels, and of value to students at all levels. These reports are one form of correlation between mathematics and English, the like of which is not met in many if any other situations. These reports deal mostly with numerical relationships and use many technical terms. They must be so written as to be understood by the receiver who is usually a stranger to the writer, and no possibility of a verbal exchange of ideas.

Another byproduct of technical mathematics courses comes from the similarity of technical and mathematical terms. There is no sharp line between these two types of terms, they shade from one to the other. The mathematics class is the only place at present that helps much to lay the foundation for understanding technical terms, and the language of handbooks. Let me quote a few terms from machinery handbooks, are they technical or mathematical? How about 'Thermal coefficient of linear expansion, mean radius, safety factor, vertical axis, diametral pitch, pitch diameter, circular pitch, helix angle, helical gear, cone gear, worm cone gear, pitch cone angle, rim speed, angular speed, tangential load, taper per foot, cutting speed, revolutions per minute, and chordal thickness of a gear tooth'? In the technical language of the automobile we hear of gear ratio, compression ratio, torque, miles per hour, pounds per horse power, cycles, and piston travel. The electrical men use such words as mil, circular mil, mil foot constant, sine curve, square root of the mean squares, thermal coefficient of resistance, and phase difference. Radio goes in for big numbers and small numbers and uses such semimathematical terms as milliampere, microfarad, micromicrofarad, wavelength, frequency, and meters per second. From the aeronautics fields come such words as dihedral angle, ratio of pounds per horse power, flat plate equivalent, angle of attack, coefficient of drag, and coefficient of lift. Practically all of the teachers of pure and applied sciences would be better satisfied with our mathematical product if the students had more familiarity with the concept of square and cubic inches, and were not such strangers to the meaning of foot-pounds, ton-miles, and the ratio of square inches of radiator surface per cubic foot of volume, and other phrases that mean mathematical operations.

Another change in industry of which mathematics teachers should be aware is the shift from fractions to decimals. Some engineers require that all dimensions be given in the decimal form. But with 12

inches per foot, 60 minutes per hour and interpolation we cannot abandon or ever neglect fractions.

Another bit of mathematics for technical students of all levels of ability, which it seems to me has been badly neglected and is in urgent need of revival, is the use of denominate numbers. This is a unit, practical for its own sake, and within reach of the students of the lower levels. I would like to call your attention also to a growing use of the metric system, nearly all the books, even the elementary ones on radio use the metric system. Look in your daily newspapers and you will find the wavelengths of the stations given in meters.

Thus far I have confined my remarks primarily to work for students of lower ability, but please do not get the impression that technical or shop work is a grand dumping place for all students of low ability. You might as well dump them all into a music course regardless of their musical ability. Lack of mechanical aptitude or technical interest is a limiting factor in a students progress in the skilled trades. Likewise lack of mathematics is a limiting factor in the progress of many a student of high mechanical ability. The real teacher of technical mathematics cannot neglect students because of their slowness, provided they can do technical work more efficiently than any other kind of work. I think I have outlined sufficient work to keep those of lower ability more profitably employed than they would be in any other subject.

To go to the other end of the scale, do not get the impression that all non college preparatory technical students are of lower ability. We get students just as capable as the best of the college preparatory, and usually they have more definite motives in their work. I have definitely in mind a number of cases where I know mathematics has provided the necessary stimulant to move these students into proceeding with college engineering.

The increasing length of college courses, the increasing demands on students in the colleges, and the uncertainty of employment will no doubt have their effect upon student demand for courses leading to employment at the end of a high school career.

There are two other industrial trends which may also be expected to affect our mathematics work in the future. First is the demand of labor unions for seniority rights. Industries will no doubt pick their future employes with greater care. One of the larger Detroit employers already requires even high school graduates, seeking employment, to take a fairly simple test in arithmetic.

Secondly industries are gradually pushing upward the age at which they will employ young men. These unemployed young men haven't much left except to go to school. If the compulsory education laws were repealed, I do not believe it would make much difference with our city schools. School efficiency seems to demand, "What is the most useful training we can give these students?"

Technical mathematics has as great or greater abundance of material for our better students as for the ones of lower ability. To better understand these possibilities let me review a few of the lines of work open to men without college training, with special emphasis on those lines where a good working knowledge of mathematics will enhance a man's chances of advancement. The following is a partial list of those lines offering employment to the larger number of men. First let me mention the metal working trades, and in connection with these, tool design and drafting, making a double process, first designing and drawing by one set of men and then the actual construction by a different set. There are considerable numbers of men employed in the steam power plants, air conditioning and refrigeration are increasing their fields of activity. Sheet metal layout, heating and plumbing will always employ goodly numbers of men. The electrical and radio fields are increasing their activities in nearly all directions,

with employment in the power plants, distributive systems, and appliances. Radio, television and electrical control devices are on the increase. In aeronautics there is construction, service and navigation. Sales of technical supplies and equipment is a line that should offer more opportunities in the future, sales people who merely know how to read the price tags and punch the cash register are not satisfactory for these positions. The best opportunities for the most capable non college technical men is in the engineering departments of the larger industries. To back my statement that there are possiblities for technical boys, let me refer to an article by Mr. Knudsen in the December Readers Digest. He says, "There is going to be more need for skilled workmen than in the past."

The widespread use of precision measurement affects also the mathematics for the more capable students. When right triangles are used, and their use is frequent, failure to interpolate for seconds will often throw the answer off by a few thousandths, hence there comes a time when interpolation must be taught. Men who are going to measure to ten thousandths of an inch must use seven figure tables, and seven figure tables practically require logarithms.

It has been my good fortune to be on the receiving end of many problems brought into Cass Technical High School by men from the industries. The newer applied sciences seem to be running into more and more uses of high school caliber mathematics. A few years ago a non college man from the Dodge Brothers plant came in with a long formula on cutting speed for a predetermined tool life in rough turning. He said he could do it all except a decimal raised to decimal power. Logarithms are of course required. Later I have found the use of a decimal to a decimal power in the following situations. The horsepower of an airplane at an altitude, iron losses due to hysteresis, amplification of sound, and the size of air ducts for air conditioning systems. Such problems call for much skill in the use of logarithms but not much theory.

My experience in solving miscellaneous problems from various lines of work has shown me another need of mathematical training not often mentioned. If you cannot do a mathematics problem you can always ask for help. But here is what I have observed. Some of the men coming to me did not know exactly what it was they wanted computed. Others did not have sufficient data. Some did not distinguish between radius and diameter. Others did not know whether their dimensions were in inches or feet. One has to know considerable mathematics just to ask an intelligent question about it, to say nothing of knowing what book to look in or what to look for or how to use it if one did happen to find the mathematics bearing on his problem.

For these better non college preparatory students who rise above the lower levels, the various trade and technical hand books offer a world of opportunities for practical problems.

I would like to mention also the slide rule. It is almost universally successful in arousing student interest, and it is quite possible to teach most of the ordinary operations before the interest lags. While the slide rule may not be usable in some types of work I look at it as one of those topics that is advantageous for a student to learn if he can.

Shop theory and handbook courses are necessarily largely mathematics but are probably better taught in connection with the various trades.

One additional major demand is made of non college preparatory students for a mathematics course that is usable. A great deal of the college preparatory mathematics can, and usually does become a mere matter of manipulation. But the successful course of usable mathematics must develop considerable power of analysis. This is another item in the training of students that is hard to develop, probably much

harder than to teach manipulation of quadratic equations. I do not feel that I know much about developing power of analysis, but I feel that the so-called "Story Problems" offer some help. I feel that a technical mathematics course must be filled with problems that do not follow a set pattern. A mixture of review problems worked in incidentally as far as possible should help in the training for analysis. It seems to me that many of our mathematics books have gone too far in classifying problems. The use of a drawing, wherever one is possible, aids in the teaching of analysis and incidentally is an aid in bringing algebra, geometry and trigonometry into closer relationship. Technical mathematics has a great opportunity for presenting problems suitable for developing power of analysis.

So many and so varied are the technical applications and opportunities that I feel that for the better students, technical mathematics can be all of college preparatory mathematics plus, and some times perhaps the plus can be the big end of the course.

A little science or technical application now and then can also be useful in keeping up interest of college preparatory students. For example if you give one of those academic problems like this one, "A digs a ditch in 12 days while B takes 20 days, how many days should it take both?", and if the students protest that this is not a practical problem, we can come back at them with, "The 12 and the 20 can be the image and object distance of a lens and the answer will be the focal length." If they don't like lenses tell them that the 12 and the 20 can just as well be two resistances connected in parallel, or if they prefer radio circuits the 12 and 20 can just as well be the microfarads of two condensers connected in series. If students do not like to solve an algebraic equation to find the distance between two cities when they could just as well look it up on a road map, it should help to tell them that the problem is only a representative of many rate velocity and time problems of mechanics, and price problems of commerce.

Technical mathematics is not an easy course to teach or to learn. It never will be an easy course, it calls for too definite a standard of accomplishment. It requires the use of many skills plus a large amount of thought, or just plain common sense, in applying the skills. Neither is it easy to find mathematics teachers with the necessary technical knowledge. The teacher training colleges do not seem to be paying much attention to this phase of mathematics. The conditions of industry demand that we accomplish definite things. Industrial changes are calling for more and more skill, more reliability and responsibility. All this leaves little room for "Horse Play" in any of the technical courses. Industry is also offering less and less to the man of no specific training.

Technical mathematics cannot and never will make super students of all. Secondly there would not be room for too many in the skilled trades, and in the third place some one must remain behind to carry on the dull monotony of the production line. There will be plenty of the college preparatory students in this line too. The majority of our high school boys in an industrial city are destined to work in the large factories. If they have any care as to in what capacity they will work it is almost necessary that they make some preparation while in high school. Technical mathematics is one of their opportunities. General education may be all right in its place but it is rather of a bread and butterless art.

A final point I want to make in favor of technical mathematics is the popular interest in science. The best evidence of this popularity was the over crowded conditions of the science exhibits at the New York Worlds Fair, while the amusement concessions ran a deficit.

As a conclusion let me summarize a few points I have observed in my teaching of mathematics to technical students. First and foremost computation must be ac-

curate, and mathematics is not useful beyond ones ability to apply it. Industry has places for men at all levels of accomplishment. The lowest level of usable attainment in mathematics is ability to copy numbers correctly. The next step to manipulate and apply fractions and decimals with unerring accuracy, followed by the use of formulas. And formulas cover the ground from simple addition, algebraic equations, negative numbers through to powers and roots, quadratic equations, logarithms, cosines etc. Mixed in among the formulas comes the necessary algebra geometry and trigonometry to support the development of the processes it is necessary to employ. Parallel to the development of formulas come other applications, extension of knowledge of terms that are partly technical and partly mathematical, and finally comes the ability to understand and use the allied sciences.

I believe it would be impossible to measure the relative advantages of technical mathematics, and the usual college preparatory mathematics. They are designed for different purposes. Technical mathematics has demonstrated its usefulness in sufficient numbers of cases to warrant its continuation and further development.

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◆ THE ART OF TEACHING ◆

Introduction of "One" Negative Number

By WILLIAM R. RANSOM
Tufts College, Massachusetts

ONE WHO HAS read the article "Introduction of negative numbers," in the October (1940) issue of THE MATHEMATICS TEACHER, with its elaboration of types of order, transfinite numbers, ordinal augends, cardinal auctors, and the "$5+\bar{3}$" may be interested to consider an alternative introduction to algebra, in which the beginner is asked to introduce only *one* negative number, -1.

Let us recognize that throughout the study of arithmetic we have been concerned with *signless* numbers, the counting numbers $(1, 2, 3, \cdots)$, the rational numbers $(2/3, 3/2, \cdots)$, the zero, and certain irrationals $(\sqrt{2}, \pi, \cdots)$ and that we know how to add, subtract, multiply and divide with these numbers, any pair with any operation giving a definite result except that we exclude division by zero and subtracting a larger number from a smaller. Let us further agree that none of this need be *un*-learned, and that we shall continue to use these signless numbers in algebra, —and let us observe (this need not be commented on to the beginner) that the numbers of arithmetic are not what certain algebraists call "positive numbers" but have been, are, and shall be still *signless*.

Algebra differs from arithmetic in that it uses letters to represent numbers, but more profoundly in that it introduces certain artifices which permit us to generalize our methods of solving problems, and to economize the attention we must give to the numerical calculations. Algebra is founded upon the augmenting of our system of signless numbers by the adjunction of three special "numbers," which it will be convenient here to call P, M, and J. They are usually denoted by $+1$, -1, and i, but it is important here to distinguish $+$ for addition from $+$ for "direction." These "numbers" are brought into problems as auxiliaries, and like the scaffolding used in building an arch, they are removed when the desired purpose has been achieved.

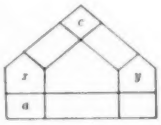
For example, here are two problems, concerned wholly with signless numbers, for which the standard solutions do not succeed if we restrict ourselves to signless numbers. *First*, $x^2+3=4x$. Certainly $x=1$ or $x=2$. But the most convenient way to ascertain this is to introduce P and M so that we can write $Px^2+4Mx+4P=P(4-3)$, whence $Px+2M=(P \text{ or } M)$, whence $Px=2P+(P \text{ or } M)$, and $x=2+1$ or $2-1$. *Second*, $(x^2+95)x=18(x^2+7)$. With beginners it is not necessary to mention J, but the present argument invites it. We note that while this problem and its solution are concerned with signless numbers only, the general method requires the introduction of both M and J. We put $x=y+6$, and $y=k(v+1/v)$, and determine $k=\sqrt{13/3}$ (signless) so that the equation reduces to $v^6+12\,k^3v^3+1=O$. From this, by completing the square, we get $(v^3+6k^3)^2=\frac{1225}{2197}M$.

It is only by putting $M=J^2$, that we can go on to find that v is $(P\cos\beta+J\sin\beta)$ where β is either 43.90°, 163.90°, or 283.90°, and finally $x=2$, 7, or 9.

To introduce the student to algebra we need not be abstract about P, M, and J,

and of course J will not be considered until the introduction is long past. We start as usual with the obviously useful idea of *pairs* of "directions," like *up and down, profit and loss*, etc.: denote "7 units up" by $7P$, and "7 units down" by $7M$, and regard $7P$ as a signless-seven multiplied by P to show that it is taken in a *standard direction* (*up* adopted as *standard*), and $7M$ as a (signless 7) multiplied by M to indicate that it is taken in the opposite direction. Thus P and M are not mere abstractions, but indicate an adopted standard and its opposite. This beginning enables us to prove their properties, not merely to assume them.

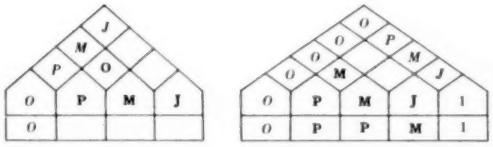
To work with numbers multiplied by P, M, and J, we need only the three fundamental laws of algebra, the commutative, associative, and distributive laws, and Tables for Addition and Multiplication. The Tables are arranged here so as not to ignore the commutative Law: the general scheme is that the elements (x or y) occupy the pentagons, underneath is the element



combined with itself ($x+x$, or xx), at a, and where their diagonals cross, at c, one element combined with another (x with y). The items in italics around the edge show that we extend to these new "numbers" these well known rules: a zero factor makes a zero product, and a number is unchanged by adding zero or multiplying by one.

Where a square is left empty in these tables, it is because the corresponding sum or product need only to be *indicated* and then dealt with by the fundamental laws. Thus $P+J$ is $+1+i$, and that is all there is about it; $P+P=2P$ by the distributive law, and is *abbreviated* $+2$; PJ can be abbreviated $+i$, and that is all there is to say about it, so it is not put into the table.

Addition: Multiplication:



For $JJ=M$, no explanation can be given: but we have seen that it has utility in the more advanced parts of algebra. For the other items in the tables, acceptable reasons can be given, based on the ideas of standard and opposite direction, the addition of zero and the multiplication by one.

Why does $P+M=O$? $P+M=O+1P+1M$, and this means that starting from zero, we go one unit in the standard direction, and then one unit in the opposite direction, thus reaching zero again: so $O+1P+1M=O$, and therefore $P+M=O$.

Why does $PP=P$? $O+PP=O+(1P)P$, which means that the one unit is measured in the positive direction, and it is measured in the positive direction, hence $O+(1P)P=1P=P$, and so $PP=P$.

The reasoning for $PM=M$ and $MM=P$ is probably suggested well enough by the two arguments given.

The fact that $a+bi=x+yi$ implies $a=x$ and $b=y$ is discovered by an inspection of the multiplication table for a product having $a-x=(b-y)i$: the only products containing i as a factor that do not contain i explicitly, are the $JJ=M$ and the $OJ=O$, the assumption that a, b, x, y do not contain i, excludes the supposition that $(b-y)i$ is JJ, and so it must be OJ with $b-y=O$.

The school-room rule that "signless numbers are positive" takes on this sensible form. Passing from arithmetic to algebra occurs when we introduce a standard direction. So while $3+5=8$ is true of

signless numbers, it is also true when we take it as $3P+5P=8P$ as we do when we are working algebraically, but we insert or remove the P's by the distributive law.

To show that $x-y$ may be regarded as $Px+My$, we say $x-y=O+x-y$, and this may be understood as "start at zero, measure x units in the standard direction, and then y units in the opposite direction," that is $Px+My$.

As for $3-5$, the bugbear of the beginner, it occurs only in algebra, and there only as $3P-5P$. By reference to the table, P appears as MM. Changing the *minus* as in the preceding paragraph, we have $3P-5P=3MM+5M=M(3M+5)=M(5-3)=M(2)=-2$. Observe that we have here nothing about "subtracting a larger number from a smaller": The only subtraction is the "five minus three," the rest of it is algebraic manipulation of the two artificial numbers, M and P, introduced to facilitate algebraic manipulation. There is never a problem in which it is necessary to subtract 5 from 3. If an islander has 3 yams, no one thinks of trying to take 5 yams from him. But the accountant, with $3P$ dollars in his favor may have to settle up with a creditor who has $5P$ dollars against him, so that a problem of $3P-5P$ may actually arise, and be solved as indicated above.

Would it not be a great gain in teaching algebra to beginners if obvious absurdities, like $3-5$, could be kept out, and the reasonable point of view that where we have a standard and its opposite, problems like $3P-5P=2M$ not only arise naturally but are solved in a sensible manner, without any suspicion of paradox? With *one* negative number instead of many, and with a clear distinction between signless numbers and numbers with a standard attribute imposed, a simple, clear, and unalarming introduction to algebra becomes possible.

The inverse operations are very simply dealt with in this approach to algebra. Addition and multiplication are directly indicated, $M+P$, or $M \cdot P$, and occasionally as in these cases the result, O or M can be taken from a table. Subtraction or division should be performed only by actually taking away an indicated addend or factor. $5=2+3$, and so $5-3$ means "take away the 3 and leave the 2." $3P-5P$ may be regarded as $(5-2)P-5P$, or $5P-2P-5P$, "take away the $5P$ and leave the $-2P$ or $2M$."

Division by zero is clearly impossible if division is treated as an inverse, or "take away" operation, $12=3 \cdot 4$ and can be divided by 3 by removing this factor; but 12 cannot be written with a zero factor and so zero cannot be divided out. Zero on the other hand, can be written as a product in so many ways, $0 \cdot 0$, $0 \cdot 1$, $0 \cdot x$, $\cdots$ that there is no telling what results if we try to divide it by zero.

Division by a fraction becomes a simple matter, if we prepare the dividend by multiplying it by *one* expressed in a suitable form. For instance, we do not try to divide a/b by x/y until we see the x and y ready to be dropped from numerator and denominator: so we write

$$\frac{a}{b} \div \frac{x}{y} = \frac{axy}{bxy} \div \frac{x}{y} = \frac{axy}{bxy} = \frac{ay}{bx}$$

For roots, the situation is simple. 4 (signless) has only one square root, 2. For signless numbers the formula $(\sqrt{a})(\sqrt{b})=\sqrt{ab}$, is correct, although it is unreliable for numbers containing P, M, or J. $4P$ has two square roots: since $4P=4PP=4MM$, the square roots are $2P$ and $2M$. Since $M=JJ=MMJJ=PPJJ$, it appears that M has *three* square roots, J, MJ, and PJ: but this anomaly must be removed *by definition*, excluding J except when accompanied by P or M. In actual experience, it appears that this is the only way we ever need to use J, it being wholly meaningless except in its relation to numbers containing P or M.

EDITORIAL

National Council Publications

THE National Council of Teachers of Mathematics has published three different types of material. First, there is THE MATHEMATICS TEACHER, the official journal of the Council, which is published every month except in June, July, August and September, the subscription price of which is $2.00 per year. Second, there are the yearbooks (sixteen of them to date) on important topics related to the teaching of Mathematics, which (except for the first and second which are now out of print) can be had postpaid for $1.25 each. Or if desired, all of the yearbooks still available, namely 3–16 inclusive, may be had from The Bureau of Publications, Teachers College 525 W. 120 St., New York, N. Y. for $14.00 postpaid. Third, the Council has published the first of a series of monographs on "Contributions of Mathematics to Civilization," which can be had from The Bureau of Publications above for 25¢ postpaid. Other monographs in the series are in preparation and will be published as soon as possible.

The Council has deliberately kept the price of the above publications down in order to make it possible for every classroom teacher of mathematics to be able to buy them. While the sales in the case of all publications of the Council have been gratifying, they have by no means reached the proportions that they deserve. No teachers' organization in this country has kept the price of its publications so low and it is hoped that more teachers will respond by purchasing these publications in the future so that the Council can continue its policy of making its publications available to classroom teachers. It is of particular importance that State Representatives of the Council and other members see to it that all of these publications are discussed at each meeting of mathematics teachers, no matter where or how small it may be.

Mathematics has an excellent chance in the years immediately ahead to regain much of its lost prestige, but only if teachers of the subject are alert and show their colors by supporting the good work that the Council is trying to do. A united effort of all concerned is the fundamental need at this time.

W. D. R.

Teacher Training

ALL of the papers in this issue (pages 99–129) inclusive were read at the annual meeting of the National Council of Teachers of Mathematics held at St. Louis, Mo. in February 1940. They have been all printed in the same issue so that they may be more available for those who wish to use them in teacher training programs or otherwise.

These papers obviously do not cover all of the aspects of the teacher training needs in mathematics, but they deal with the more essential features of the program. They should be helpful also to all those who are interested in the work on Teacher Education that is now being done by the Commission on Teacher Education of the American Council on Education.

W. D. R.

◆ IN OTHER PERIODICALS ◆

By NATHAN LAZAR
The Bronx High School of Science, New York City

American Mathematical Monthly

June–July 1940, Vol. 47, No. 6

1. Blumenthal, L. M., "A Paradox, a Paradox, a Paradox a Most Ingenious Paradox," pp. 346–353.
2. Grossman, H. D., "A Generalization of the Water-fetching Puzzle," pp. 374–375.
3. Brown, A. B., "To Text-Book Writers—and Readers," pp. 375–381.
4. Moulton, E. J., "Comments on the Preceding Paper," pp. 381–383.

August–September 1940, Vol. 47, No. 7

1. Blumberg, Henry, "On the Technique of Generalization," pp. 451–462.
2. Cameron, E. A., "A Program in Freshman Mathematics Designed to Care for a Wide Variation in Student Ability," pp. 471–473.

October 1940, Vol. 47, No. 7

1. Keller, M. W., Shreve, D. R., and Remmers, H. H., "Diagnostic Testing Program in Purdue University," pp. 544–548.
2. Hopper, Grace M., "Mathecinematics," pp. 565–568.

November 1940, Vol. 47, No. 8

1. Fort, Tomlinson, "Mathematics and the Sciences," pp. 605–612.
2. Scheffé, Henry "At What Level of Rigor Should Advanced Calculus for Undergraduates be Taught?", pp. 635–640.

National Mathematics Magazine

November, 1940, Vol. 15, No. 2

1. Sewell, W. E., "Logarithmic Properties of a Power of x," pp. 55–60.
2. Yates, Robert C., "The Angle Ruler, the Marked Ruler, and the Carpenter's Square," pp. 61–73.
3. Richeson, A .W., "Hypatia of Alexandria," pp. 73–82.
4. Finkel, Benjamin F., "A History of American Mathematical Journals," (continued), pp. 83–96.
5. Herbert, Harriet B., "The Tesseract, $(a+b)^4$, a Demonstration of the Binomial Theorem in Fourth Dimensional Geometry," pp. 97–99.

School Science and Mathematics

January 1941, Vol. 41, No. 1

1. Williams, Sidney J., "Science, Mathematics and Safety," pp. 2–6.
2. Coffing Esther A., "The Relationship between Silent Reading Ability and Arithmetical Ability," pp. 10–14.
3. Christofferson, H. C., "The Crucial Place of Seventh and Eighth Grade Mathematics in Education for Competency," pp. 55–62.
4. Rosser, J. Barkley," Many-Valued Logus," pp. 99–100.

Miscellaneous

1. Buswell, G. T., "Selected References on Elementary-School Instruction: Arithmetic." *Elementary School Journal*, 41: 210–212, November, 1940.
2. Cleaveland, B. B., "Number Seat Work," *American Childhood*, 26: 26, December, 1940.
3. Dimmitt, R., "Modern Objective Tests: Number Test." *Grade Teacher*, 58: 58, December, 1940.
4. Dwyer G. W., "Tests for Middle and Upper Grades: a Christmas Arithmetic Tests," *Instructor*, 50: 50, December, 1940.
5. Fish, G. L., "Teaching Family Life through Arithmetic." *Journal of Social Hygiene*, 26: 317–319, October, 1940.
6. MacQuarrie, A. P., "Motivation and Illustration" *School* (Secondary Edition), 29: 260–263, November, 1940.
7. Rue, F. C., "Number Work," *American Childhood*, 26: 27, December, 1940.
8. Tessin, L. D., "Number Seat Work," *American Childhood*, 26: 28, December, 1940.
9. Tully, M. and Fraden, J. H., "Correlation of Mathematics with Science," *California Journal of Secondary Education*, 15: 352–354, October, 1940.
10. "What is Wrong with Mathematics? Current Teaching Examined." *Times Educational Supplement* (London) 1330: 411, October 26, 1940.

BOOK REVIEWS

Mathematics in the Modern World. By Albert E. King, George L. Paley, and George W. Patterson. Houghton Mifflin Co., 1940. Book One A (seventh year—first half), vi+282; Book One B (seventh year—second half), vi+278. Price, $.80 each.

These two books are the first of a new series of junior high school textbooks based on the New York City Course of Study and Syllabus in Mathematics for the 7A–9B, as approved in 1939 by the Board of Superintendents for Experimental Study. It goes, however, beyond the recommendations of the syllabus both in the excellent material provided for bright classes as well as in the emphasis placed on many important topics.

The style of writing should appeal to the youngsters as it is conversational in tone and uses dramatization to introduce new topics. There is an abundance of illustrative material and practical problems, a chapter on fundamental operations, and another chapter on remedial work in fundamental operations and in the solution of problems. The exercises are carefully graded, objective tests are frequently given while summaries and vocabularies are placed at the conclusion of each topic.

Book One A begins with informal geometry and gives the pupil a feeling of beginning a new branch of mathematics rather than continuing over familiar arithmetic. The approximate nature of measurement is stressed as well as the concept of the degree of accuracy in measurement.

The type is clear, the photographs attractive and relevant, and the binding substantial.

The progressive teacher of junior high school mathematics should not fail to get acquainted with these harbingers of a new series.

NATHAN LAZAR

The Fundamental Equation of the Construction of the World. By Lawrence Bruehl. Prosperity Publishing Company, New York, 1938. 24 pp. Price, $.25.

The Mathematics of Unlimited Prosperity. By Lawrence Bruehl. Prosperity Publishing Company, New York, 1939. 405 pp. Price, $4.00.

The first book has a subtitle: "The Analysis of the Human Mind—A Treatise on the Mathematical Solution of All Economic, Political, Religious, and Scientific Problems." The second book has the following subtitle: "The Triumph of Man over Nature—the Basic Solution of All Scientific, Religious, Political, and Economic Problems."

Chapter one of the second book begins as follows: "This book is not 'a' book on economics, it is 'the' book on economics which will put all professors of economics on the spot. This book is for the economic behavior of human society what the Bible is for Christendom. There is no compromise."

Although there is danger in quoting out of context, there is often no better way of imparting the fundamental flavor as well as the basic ideas of a book.

Chapter 52 is entitled "The Greatest Capitalist or the Master of Disintegration" and begins as follows: "There is only one real capitalist in existence. This capitalist is God as the unlimited master of disintegration. God is the highest and greatest capitalist, and there are no other capitalists beside him. All other capitalists are fakers and burglars, who disregard the Second Commandment of God. The predominant fakers and burglars are the robber barons of high finance who operate from their pigeon-hole the Federal Reserve System."

The books are a conglomeration of theology, economics, politics, mathematics, and metaphysics. There are few questions to which no answer is given, and there seem to be no problems to which the solutions are not readily available.

NATHAN LAZAR

Exercises in Reasoning. By Joseph A. Nyberg. Published by the author at 10505 South Bell Ave., Chicago, Ill., 1940. 84 pp. Price, single copies, $.50; 10 or more, $.30 each.

It is a commonplace that geometry is taught in the secondary schools for other purposes as well as for the mathematical facts it contains and for the mathematical patterns of reasoning it illustrates. What makes geometry an important, almost crucial, subject in the curriculum is the possibility of using its facts and patterns as prototypes for those found in other fields of human endeavor.

Unfortunately very few of the writers of textbooks have taken the trouble to insert such non-mathematical material in their books, with the result that the conscientious teacher who wants to do more than lip-service to his ideals is in a quandary. The few books that do contain such exercises suffer from a lack of complete integration of the geometric and non-geometric

exercises. Moreover, they often contain sequences of theorems which the experienced teacher refuses to accept. As a result he continues using his familiar textbook and sequence, and thus sacrifices or, at best, does little justice to those objectives which he himself considers the *raison d'être* of geometry.

The excellent booklet, *Exercises in Reasoning* is designed to meet the needs of such teachers and their pupils. It may be used with any textbook or sequence of theorems. Geometric concepts, propositions, exercises and transformations are introduced primarily for the light they shed on their non-geometric counterparts. Exercises are chosen from the sciences, sports, history, and current events, and are presented in a manner that will interest, even beguile, the adolescent boy and girl.

The range of topics treated will become clear from the following list of chapter headings: general statements; how general statements are used; definitions, assumptions, proofs; geometric proofs; converses; the indirect method; inverses; contrapositives; necessary and sufficient conditions; inductive, scientific, and deductive methods; mistakes in reasoning; problem solving; Euler's circles; propaganda.

Unfortunately, there are some defects which should be pointed out. The treatment of converses and inverses is limited to theorems having one conclusion, though no limit is set to the number of statements in the hypothesis. The contrapositive is limited to statements having only one condition in the hypothesis and one conclusion, thus robbing it of its potential application to the great number of theorems having more than one condition in their hypothesis.

The treatment of the Indirect Method of proof is not quite convincing. The relation between contraposition and indirect proof is not pointed out.

These and other shortcomings should not, however, be allowed to eclipse the fundamental fact that this booklet is a pioneering effort in a much neglected field and should be in the hands of every progressive teacher of mathematics. It would be still more useful if it were put in the hands of every student of geometry as a constant companion to his textbook.

NATHAN LAZAR

Introductory Mathematical Analysis. By Joel S. Georges and Jacob M. Kinney. The Macmillan Company, 1938. XIII+595 pp. Price, $3.00.

This new book presents, in an admirable way, the mathematical concepts and processes found in the traditional courses in the first two years of college mathematics.

Having as its core the concept of function, the authors, by considering the polynomial, the linear functions of two and three variables, implicit quadratic, exponential and circular functions, have been able to bring out the importance of the function concept as well as its meaning. Every opportunity is taken to put the function to practical use. Problems from various fields such as business, biology, chemistry, physics, astronomy and economics are embodied in the text thus affording a wealth of material from which the teacher as well as the student can select. In this way not only are the needs of the general student taken care of but also those of the one who may wish to become a specialist.

Not the least of the book's attractiveness is the interesting style in which it is written. A satisfactory appeal to the eye is also made by fine binding, typography and numerous excellent drawings.

The use of this book should insure a maximum of understanding and a minimum of confusion on the part of the student and to the teacher a concise means of imparting the important concept of function.

E. J. SCOTT

Science in a Tavern. By Charles S. Slichter. The University of Wisconsin Press, 1938. ix+186 pp. Price, $3.00.

This very interesting volume from the pen of one of our well-known teachers in a leading State University will be of interest to all scientific men and women. It is a collection of essays which have appeared from time to time in one way or another.

The following table of contents will give an idea of the possibilities of this interesting "by-product in the life of a busy teacher":

SCIENCE IN A TAVERN

The Royal Philosophers
The Club

SOME MEN OF SCIENCE

Polymaths: Technicians, Specialists, and Genius
The Principia and the Modern Age

THE MEANING OF SCIENCE

Science and Reality
Industrialism
Science and Authority

TO YOUTH

The New Philosophy
The Self-Training of a Teacher
Heaven's Highway

W. D. R.

The Secondary School. By Charles Watters Odell. The Garrard Press, 1939. viii + 606 pp. Price, $3.00.

Dr. Odell is Associate Professor of Education at the University of Illinois. The book he has written might well be designed as a text for his course in principles of education. He declares his purpose not to discuss psychology of education or teaching methods but to treat of the development of secondary education in the United States and elsewhere; its present status, "its relationships to the portions of the school system below and above it; its objectives or functions; curriculum construction; the extra-curriculum; pupils, particularly their individual differences and the characteristics of adolescence; guidance, educational, vocational, personal and whatever else it may be; the teacher; and certain phases of organization, administration, and public relations."

All this is for the inexperienced undergraduate preparing himself for teaching, not for mature and seasoned teachers looking for something new. It is descriptive rather than doctrinarial. The following statement probably expresses better than any other the spirit of the book: "Each student should do his own critical, constructive thinking, basing it upon both the best evidence and the best educational, philosophical, psychological, sociological, and scientific thought. In many cases there probably is not, and never will be, any single specific solution that is better than all others, but rather a small group of better ones as opposed to a group of inferior ones.

"The student who has heard much of the recent emphasis on scientific investigation and objective methods in the field of education may be disappointed by the statement made in the preceding paragraph, but he should reconcile his thinking to this fact. In a relatively few cases such methods have given definite answers; in many they are contributing something of value; in numerous others they will be of assistance; in still others they will avail little. Some of the most fundamental problems relate to the philosophy of life, and are not capable of solution in this manner. Many others may perhaps be solved, or at least have their solutions greatly aided, by such methods; but the advances required before this can be accomplished are so great that the promise of help in the near future is too slight to afford much hope or to justify waiting until it arrives."

The thirty-five chapters of the book are grouped into six parts: I. Foreign and American Systems of Secondary Education, II. Our Secondary School Population, III. The Secondary School Curriculum, IV. The Secondary School Extra-Curriculum, V. Articulation of the Secondary School with Other Schools and with Society, VI. The Secondary School Teaching Staff.

W. D. R.

Evolution of Factoring. By Hobart Franklin Heller. Keystone Publishing Company, 1940, vi and 165 pages. Price, $1.75.

All who are interested in the teaching of algebra should read the thrilling story "The Evolution of Factoring" as described by Dr. Heller.

At first one may be annoyed at the apparently irrelevant material presented in the introductory chapters, but the lag of interest will not persist. Before one has reached the last chapter, the seemingly irrelevant material fades into the background, only to give a richer setting to the fascinating story, "The Evolution of Factoring." It is a technical volume of research but its story is so strange that it seems like a fairy tale.

When one has read this study, he will ask himself such questions as: "Is it possible that factoring began with one example that was accidentally placed in a textbook? Is factoring an outgrowth of this example being imitated and expanded, without any evidence of its value, until in the days of Wentworth hundreds of such examples appeared?" The research of Dr. Heller tends to support such a hypothesis.

Perhaps Dr. Heller goes too far when he suggests the possibility of the factoring included in the calculus being copies from the algebra textbooks, but who knows?

Dr. Heller concludes further, as a result of his study, that Bonnycastle and Van Schooten, who are given only scant recognition by historians in the field of mathematics, were two men whose work had a very great influence in determining the content of the elementary algebra in the nineteenth century. Dr. Heller tends to minimize the supposedly great French influence upon the teaching of algebra in the United States. He contends that the most important source of influence upon the content of American textbooks of algebra in the nineteenth century was the English textbooks.

His study should encourage teachers of elementary algebra to eliminate difficult types of factoring or, at least, to reconsider the value of factoring before devoting considerable time to such a unit.

KENNETH BROWN

The Decline of Mechanism in Modern Physics. By A. d'Abro. D. Van Nostrand Company, 1939. 982 pp. Price, $10.00.

The educated layman who has read with uneasiness or relief of the passing of a mechanis-

tic and cocksure science with the turn of the century will be attracted by the title of this book and by the purpose stated in the preface: "to give a semi-popular presentation of the progressive growth of the new ideas, starting from the most elementary notions and giving due consideration to the mathematical development without which the new theories could never have been constructed." The major peaks in the new twentieth century theories with which the book deals are the Theory of Relativity and the Quantum Theory.

According to the preface, the non-mathematical reader should have no great difficulty in following the presentation. Semi-popular the treatment may be, and, so far as vocabulary and simplicity of description are concerned, it is. But the concepts underlying these theories are so difficult that a simple minimal explanation can suggest no more than the shadowy outlines of a mystery, even to the upper ten percent of college-educated persons. There are Ph.D.'s in mathematics and the physical sciences who would have to read d'Abro's book with more of respectful attention than critical judgment.

Knowing that the austere heights exist, even without hope of approaching them, is worth while to many persons. The text contains fascinating qualitative ideas like the following: "We conclude that the mechanistic theories of the past should be viewed as first approximations, and that subsequent discovery has served to refine but not altogether to destroy them. Furthermore, dogmatic assertions on the finality of the modern theories are out of place. For all we know, the modern theories in turn may be only approximations which will appear strangely crude twenty years hence. The field theories, in particular, cannot be the last word—they may be accepted only provisionally till a higher stage of approximation is reached. The progress of science must be viewed as the forging of a chain; each link has its place and cannot be suppressed without breaking the continuity of the chain; and the mechanistic theories represent one of these indispensable links."

The organization of the book is such that its easier descriptive parts, coming first, may be read, and the more technical chapters XXXI to XL, "not essential to a general understanding of the new ideas, . . . may be omitted in consequence."

W. D. R.

General Mathematics Workbooks, Books 1, 2, and 3, for Grades 7, 8, and 9, by William David Reeve. The Odyssey Press, Inc., 1940. 185 pp. each. Price $0.64 each.

The appearance of this series of workbooks is of importance to teachers of mathematics for at least three reasons: First, it marks the debut of Professor Reeve as the *sole* author of textbooks wherein he can give free rein to his progressive spirit tempered only by his long pedagogic experience. Second, these books follow (perhaps "anticipated" is the more correct word) the recommendations of the recently published *Fifteenth Yearbook*, prepared by the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics, of which the author was an active member. Third, they set up enviable standards for content and method as well as for typographical clarity and attractiveness.

Book 1 is intended for the seventh year and is organized around the following central themes: Fundamental geometric forms; elementary drawing and measurement; fundamental geometric constructions; symmetry, congruence, and similarity; information graphs; geometry of position; the language of size; fundamental processes with whole numbers; measures used in daily life; fundamental processes with common fractions and decimals; the application of percentage to the problems of the home and community, business, and saving and banking. The mere enumeration of these central themes cannot possibly describe the pedagogic subtlety with which they are interwoven, and the variety of types of questions that are used throughout the book. Some pages are designed to stimulate interest, others to maintain it throughout a given topic, and still others to point to valid mathematical conclusions.

In Book 2 the concepts of Informal Geometry covered in the previous year are reviewed and their meanings extended to include some more difficult geometric concepts and applications. The Language of Algebra which was begun in the previous year is then reviewed and considerably extended to include the formula, the equation, the graph and directed numbers.

Arithmetic is not forgotten, however, for there is an extended treatment of basic arithmetic skills, followed by their application to those topics of business arithmetic which are of interest to students of the age for which this material is intended.

The boys and girls with whom the reviewer used Book 2 last summer were especially pleased at how easily they were introduced to what they feared were difficult topics. They also expressed keen delight at the numerous drawings, appropriate photographs, and half-tones with which the book abounds.

Book 3 is devoted in the main to those parts of algebra which an intelligent member of any modern community should know. Obsolete concepts, processes, and manipulations are, of course, omitted. It also contains an interesting

section on Numerical Trigonometry with emphasis on the importance of indirect measurement. Following the recommendations of the *Report* a simple treatment is included on Demonstrative Geometry. The primary purpose of this is to acquaint the student, to whom the ninth year may be his last contact with mathematics, with the nature of deductive reasoning.

Throughout these books emphasis is laid upon the individual, unique nature of each of the branches of mathematics that are presented. Since each is developed in consecutive treatment the teacher can easily locate the material which he may wish to use at any time for instruction or testing. But equal emphasis is given to the inculcation of the idea of the unity of mathematics by the inclusion of instruction and tests that involve operations common to all the branches.

Each of the books contains more than one hundred and fifty tests. These have been carefully graded and constructed so that they may be used either as a teaching device or as a testing instrument. Throughout the books there is constant emphasis on understanding the operation performed, on making the student conscious of the thought processes and mathematical principles underlying the work done, and on knowing the meaning of the reasons for the steps taken.

Together with the insistence on meaning and understanding the author maintains a freshness in the treatment of the topics by the inclusion of only those mathematical materials that relate to the everyday experiences and interests of the pupils. It is only by such a dual approach that elementary mathematics can ever hope to occupy the position in general education which is rightly hers.

NATHAN LAZAR

A Study of the Number Concept of Secondary School Mathematics. By Howard Franklin Fehr. Edwards Brothers, 1940. Pages 111 and 202. Paper. Price $1.60.

Since the concept of number is one of the basic ideas needed in the understanding of mathematics, and since added emphasis is being placed on the teaching of mathematics for meaning rather than mechanical manipulation, the work of Dr. Fehr is a timely addition to the field of literature for teachers of secondary mathematics.

In this study, Dr. Fehr presents the historical development of the various forms of numbers from the earliest written records to the present time. After a critical examination of textbooks and a careful analysis of answers that teachers give in regard to their conception of number, he emphasizes the need for a better understanding of this basic concept in the teaching of mathematics. He says, "In order that teachers may gain a fundamental background of the concept of number, on which all secondary school algebra can be based, in order that all statements and processes given in current textbooks can be intelligently appraised, it is necessary that some elementary logical development be presented." At some length he develops a logical system of natural, fractional, rational, real and complex numbers. This presentation should be of interest not only to the mathematics teacher, but also to the student of pure mathematics.

The study answers many questions such as the following: What is a number? What is the modern conception of $\sqrt{-1}$? What is a number system? What is function as interpreted in pure mathematics? What criterion is used in establishing mathematical laws? What concept of a variable is required in modern mathematical thought?

Dr. Fehr concludes his study in a very appropriate way, and one which should be of benefit to mathematics teachers. In his final chapter, he gives suggestions in regard to inculcating the basic concepts of a number system into mathematical instruction for secondary school pupils.

K. BROWN

The Place of Mathematics in Secondary Education. The Final Report of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics. Teachers College, Columbia University, New York: 1940.

Ever since the publication of the report of the National Committee on Mathematical Requirements in 1923, which recommended further experimentation with the mathematical curriculum, teachers have been looking forward to a new report which would continue the work of that Committee. The task was taken over by the Joint Commission, and the findings are now reported in the *Fifteenth Yearbook* of the National Council of Teachers of Mathematics.

It is not the intention of the commission to propose a curriculum to be followed rapidly by all types of schools, but rather to submit a plan around which, with proper modification and rearrangements, individual schools may build a mathematics program suitable to their specific needs. The reader will agree that the commission has been equal to the task, and that a great deal will be accomplished if teachers of mathematics

will follow the advice and suggestions presented in the report.

The reader will be impressed from the start with the fact that the report is the outcome of a large amount of study and research, and does not merely reflect the opinions of one or several members of the committee. To understand the proposed plan thoroughly, it will be necessary for the teacher to study carefully the first chapters leading up to the proposed plan.

These chapters call attention to the need of modifying traditional curricula to meet the changing conditions of our time. It is pointed out that such modification must be based on adequate correct ideals and principles. Hence a comprehensive view is given of education as a whole, and a detailed discussion is presented of broad and general objectives which will guide the teacher in selecting and organizing materials for teaching purposes.

Objectives are classified in two ways: First, those having a factional and impersonal aspect, such as the acquisition of facts, skills, concepts, and general information; and second, those having personal and psychological bearing, such as habits of work, attitudes, interests, insight, modes of thinking, types of appreciation, and creativeness. The report deals fully with the objectives of the second type, and shows how mathematics may contribute to these objectives of general education. It leaves the reader with the conviction that mathematics deserves a place in modern secondary education.

After these important preliminary discussions, the report proceeds to the formulation of the mathematical curriculum. Starting with a list of guiding principles to direct the selection and organization of instructional materials, the curriculum is expressed in terms of seven major fields: number and computation; geometric form and space perception; graphic representation; algebra and trigonometry; logical thinking; relational thinking; and symbolic representation. Each field is subdivided into five categories which receive detailed consideration. Finally, the work for each grade is presented in descriptive form. To give the reader a quick view of the entire plan a grade-placement chart is attached in Appendix VI.

The plan just described is followed by a second, not worked out with as much detail as the first, but conforming to the principles of curriculum building laid down by the commission.

A later chapter adds two plans for a junior college mathematics course.

About one-half of the report of the commission is concerned with the curriculum. The other part deals with problems of vital interest to all teachers of mathematics, such as the problem of the retarded and accelerated pupil, the evaluation of pupil progress and the education of teachers of high school and junior college mathematics.

Of no less importance are the chapters presented in the Appendix, which contain discussions of the mathematical needs of pupils and adults, a review of the studies related to transfer of training, the commission's position in regard to the meanings of important mathematical terms and symbols; and the equipment of the mathematics classroom.

The report of the commission should appeal to the following groups of people: teachers of mathematics, administrators of secondary schools; students of education, and curriculum experts.

E. R. Breslich

Elementary Algebra. By Aaron Freilich, Henry H. Shanholt, and Joel S. Georges. Silver Burdett Company, 1940, vi+544 pages. Price $1.36.

This new book is written by three men, each of whom has had a most useful type of classroom experience. This experience has enabled them to discover many of the handicaps which pupils encounter in the learning process and has made it possible for them to write a book based on experience that pupils can easily read and master. The vocabulary is simple, though adequate; meanings are easily acquired, so that the pupil has only to develop the techniques related to the use of the symbolism involved in such a course.

The algebraic work is based upon the pupil's previous knowledge of arithmetical principles, and the work is thus extended so as to give the pupil a real understanding of what algebra means. Too often pupils go through an entire year of algebra without knowing what it means. An intelligent use of this book should obviate such a result.

The authors have provided for individual differences among pupils by carefully grading and arranging the exercises and problems in three groups A, B, and C. The A group contains material considered fundamental for all. The B group is intended to furnish the teacher with material from which he may select those parts adapted to meet the varying interests and abilities of pupils. The C group contains material advanced enough to challenge the ability and the demands of the most gifted pupils. Unless three separate courses in algebra are written to meet the needs, interests, and abil-

ties of at least three different types of pupils, textbooks in algebra must make some such arrangement as that above.

W. D. R.

Graphs: How to Make and Use Them (Revised Edition). By Herbert Arkin and Raymond R. Colton. Harper and Brothers, 1940, xvi+236 pages. Price $3.

This is a completely revised edition of an earlier manual on graphical methods. It covers nearly all of the types of graphic technic in present day use, and by means of many illustrations shows how statistical information of all kinds should be presented so as to be most intelligible to the reader. The table of contents runs as follows:

I. The Principle of the Graph
II. Construction of the Graph
III. Graphic Layout
IV. The Equipment for Graphic Presentation
V. The Line Graph
VI. The Ratio Chart
VII. The Bar Chart
VIII. Area and Solid Diagrams
IX. Graphing Relationships
X. Map Graphs
XI. Organization Charts
XII. Graphs for Computation
XIII. Control Charts
XIV. The Reproduction of Graphs
XV. Statistical Tables and Reports.

All mathematics teachers in the secondary school will want to see this new book.

W. D. R.

Mathematics Instruction in the University High School. By members of the Department of Mathematics of the University High School of the University of Chicago. Publication No. 8 of the Laboratory Schools of the University of Chicago, Department of Education, 1940, xiii+184 pages. Price $1.00.

This volume is dedicated to Professor Ernst R. Breslich, who served the University High School as teacher and as Chairman of the Department of Mathematics from 1904 to 1939.

The preface of the book states that "the progress made in the organization of the curriculum in mathematics and its teaching in the University High School of the University of Chicago is so closely identified with the career of Ernst R. Breslich that a review of the present status of secondary school mathematics in this institution is largely a review of his professional work."

It is gratifying to see an institution recognizing the work of one of its most faithful members in such a way during his lifetime, and the publication must be a source of satisfaction to Professor Breslich, who well deserves all the recognition they have given him.

The best way to indicate the nature of the contents of this new book is to give the various chapter titles:

I. The Development of the Mathematics Curriculum in the University High School.
Purposes of the Laboratory Schools. The Pupils of the Laboratory Schools. Experimentation with Methods and Materials in Mathematics.

II. The Aims of Teaching Mathematics in the University High School.
The Importance of Setting Up Clearly Defined Aims. Mathematics Contributes to the Educational Objectives. General Mathematical Objectives. Course and Unit Objectives. Relation of the Aims of the Teaching of Mathematics to Teaching Procedures.

III. Development of a Program in Mathmatics in which the Courses are Correlated with Each Other and with Other School Subjects.
Difficulties Experienced with Traditional Courses. Correlating the Courses in Mathematics. The Correlation of Mathematics and Science.

IV. Description of Courses by Units.
The Selection and Organization of Materials of Instruction. Outline of Units for the Seventh, Eighth, Ninth, Tenth, and Eleventh and Twelfth Grade Courses.

V. Teaching Procedures.
Unit Organization of Mathematics Courses. Steps in Teaching a Unit. Unit Test. Provision for Individual Differences. Remedial Program.

VI. Illustrations of the Procedure in Teaching a Unit.
A Unit on Quadrilaterals. A Unit on Problems Leading to Linear Simultaneous Equations.

VII. The Testing Program.
The Unit Test. Course Test on Minimum Essentials. Standardized Achievement Tests.

VIII. Special Instruction in Arithmetic.
Necessity for Special Instruction in Arithmetic. Organization of Special Classes. Disadvantages of Special Classes.

IX. Program Enrichment for Classes in Mathematics.
Provision for Individual Differences. The Mathematics Library. Supplementary Projects for Superior Pupils. The Mathematics Club.

X. The Relation of Mathematics to Other Subjects in the Curriculum. Mathematics and Physical Science; and the Social Studies; and the Practical Arts; and the Fine Arts. Conclusion.

W. D. R.

The Dozen System, An Easier Method of Arithmetic. By Terry, George S. Longmans Green & Co. 1941. 53 pp. Heavy paper covers.

Practically all of the advanced arithmetics of the middle of the last century contained a section or chapter dealing with duodecimals. As long ago as 1866, Thomas Leech in *Dozens versus Tens* advocated twelve as a better base than ten for a number system. Mr. Terry's *Duodecimal Arithmetic* (1938), and the present volume, represent a definite attempt to prepare adequate texts for the teaching of the duodecimal system. The present volume contains illustrations and a few exercises in addition, subtraction, multiplication, division, fractions, factors, powers, reciprocals, and roots. There is a duodecimal slide rule, a duodecimal circle. Commercial applications are shown. New tables of weights and measures are developed.

All mathematicians will no doubt agree that Mr. Terry has rendered a great service to pure mathematics. An enthusiast in the field of mathematics will enjoy this volume exactly as a "best seller" enthusiast welcomes *Fielding's Folly* or *Oliver Wiswell*. But in either case there are no practical values. The failure of the metric system as a system (even though enforced by heavy penalties) shows that academic minds are capable of conceiving beautiful systems which at the same time can be most impractical. As the reader works through Mr. Terry's book, he might keep in mind that a change to a duodecimal system of numbers would be no more difficult than the change to the metric system of weights and measures has proven to be since it was first launched by the French Constituent Assembly one hundred forty-eight years ago.

Guy M. Wilson
Boston University

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